

To extend the authors' characterization of u_r , the radial velocity through a turbine rotor: it is, to use a colloquial expression, “the elephant in the room” of wind turbine aerodynamics. There is little theoretical guidance on possible ways of determining it, partly because the one-dimensional analysis that leads to the Betz-Joukowski limit does not give the length of the near-wake. One of the few theoretical guides to the magnitude of u_r is the result proved for a purely-radial actuator disk by Limacher & Wood (2021, additional reference below):

$$\int_0^\infty (a^2 - u_r^2) r \, dr = 0 \quad [1]$$

where the axial induction factor $a = 1 - u_x$; u_x is the axial velocity, and all velocities are normalized by the freestream. r is the radial co-ordinate. In words, the area integral of the square of the axial induction factor equals that of the square of the radial velocity. Equation (1) was shown to be accurate for actuator disk analysis in Chapter 7 of van Kuik (2022). It is reasonable to assume, therefore, that the order of a is that of u_r in general, although Wood & Limacher (2021) showed u_r is smaller than a near the axis (where u_r must be zero) and larger near the outer limit of the actuator disk. The elephant, therefore, is substantial.

It is also reasonable that any model for the radial velocity should be consistent with Eq. (1) even though its application to a coned rotor with a finite number of blades requires further analysis. I note that neither model considered in the note makes a comparison to Eq. (1) nor provides sufficient information to allow one to be made. One further comment: vortex models for the induced velocities are important as u_r is determined entirely by the trailing vorticity for a purely-radial rotor. These models require the vortex circulation, which in the note and in the references, is determined entirely by the lift of the blade element. This is incorrect, because, as pointed out by Glauert (1934, p217), the blade element drag reduces the circulation in the equation for conservation of angular momentum applied immediately behind the rotor.

Additional References

Glauert, H. (1983). *The Elements of Aerofoil and Airscrew Theory*. Cambridge University Press, 2nd edition.

Limacher, E. , & Wood, D. (2021). An impulse-based derivation of the Kutta–Joukowski equation for wind turbine thrust. *Wind Energy Science*, 6(1), 191-201.

van Kuik, G. (2022). *The Fluid Dynamic Basis for Actuator Disc and Rotor Theories: Revised Second Edition*. TU Delft Open Publishing.

Wood, D., & Limacher, E. (2021). Some effects of flow expansion on the aerodynamics of horizontal-axis wind turbines. *Wind Energy Science*, 2021, 1-21.