



Brief communication: A fast parabolic RANS numerical model with analytical pressure closure for continuous induction-to-far-wake prediction in wind farm flows

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Abstract. Parabolic Reynolds-Averaged Navier-Stokes (RANS) approaches offer an attractive compromise between the accuracy of RANS and the efficiency of analytical methods. However, due to neglecting pressure gradients, these models often require near-wake corrections. Moreover, they are typically based on over-simplistic turbulence closures. To address these limitations, this brief proof of concept couples a recently developed analytical pressure-gradient formulation (Bastankhah et al., 2026) with a parabolic RANS framework. It also uses the $k-\varepsilon-f_p$ turbulence closure, well suited to strong near-wake velocity gradients (van der Laan et al., 2015). The model closely reproduces RANS results, while reducing per-core computational costs by more than three orders of magnitude.

1 Introduction

Wind farm flow modelling requires a balance between physical fidelity and computational efficiency. Large-eddy simulation provides detailed wake dynamics but remains computationally expensive, while analytical wake models are efficient but rely on simplifying assumptions. Parabolic Reynolds-Averaged Navier-Stokes (RANS) approaches offer a middle ground, retaining wake superposition and advanced turbulence closures at substantially lower cost than full RANS. Several parabolic formulations have been proposed (Ainslie, 1988; Iungo et al., 2018; Martínez-Tossas et al., 2019; Bradstock and Schlez, 2020; Cheung et al., 2025; Klemmer and Howland, 2025). However, they commonly neglect the streamwise pressure gradient near the wind turbines, which limits their ability to represent the induction and near-wake regions, which in turn govern the rest of the wake. Overly simplified turbulence closures (e.g., zero-equation and one-equation models) may also restrict the representation of turbulence evolution, particularly in regions of strong velocity gradients.

Here, we develop a continuous Forward Marching Reynolds-Averaged Navier-Stokes (FM-RANS) framework that resolves the flow from upstream of the turbine through the induction, near-wake, and far-wake regions. The streamwise pressure gradient is introduced using the analytical actuator-disk pressure solution of Bastankhah et al. (2026), while turbulence is modelled using the nonlinear $k-\varepsilon-f_p$ closure, well suited to handle strong velocity gradients in the near-wake region (see Apsley and Leschziner (1998); van der Laan et al. (2015)). As a proof of concept, the methodology is assessed against full RANS simulations of isolated turbines and four-turbine configurations under neutral atmospheric conditions. Throughout, quantities are



time-independent and Reynolds-averaged, with $u_j = (u, v, w)$ the mean velocity components along $x_j = (x, y, z)$; the subscript
 25 h denotes hub height. Turbulence intensity is defined as $I_k = \sqrt{2k/3} / \|u_i\|$.

2 Reference RANS simulations

Reference RANS simulations were performed using OPENFOAM-v2406, with the $k-\varepsilon-f_p$ turbulence model and the actuator-disc implementations from ACTUATORDISKFOAM (Baungaard et al., 2025) (<https://github.com/mchba/actuatorDiskFoam>). Two configurations are considered: an isolated wind turbine and four aligned turbines with a streamwise spacing of $7D$.
 30 Boundary conditions consist of a rough-wall ground, a free-slip top boundary, and periodic lateral boundaries. The hub-height turbulence intensity is varied through the ground roughness. Inflow profiles, shown in Fig. 1, are generated using a single-column model. The turbines were represented by uniformly loaded actuator disks with diameter $D = 240$ m and hub height

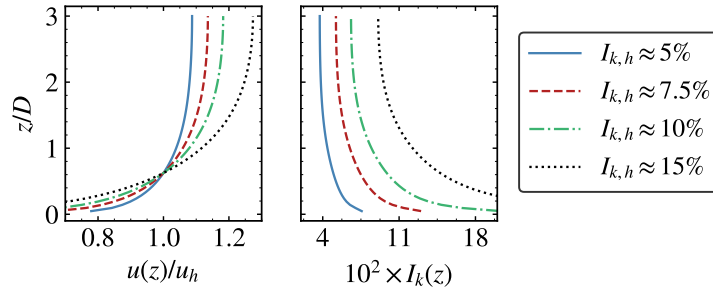


Figure 1. Vertical profile of the normalised wind velocity magnitude and turbulence intensity.

150 m. The baseline thrust coefficient is $C_T = 0.8$, with one $C_T = 0.5$ case. The computational domain size is $(48D \times 16D \times 3.125D)$, with 16 cells per rotor diameter and the first rotor located $8D$ downstream of the inlet. Simulations were considered
 35 converged when all transported-variable residuals fell below 10^{-6} , or after 4000 iterations. In the latter case, all residuals are below 10^{-5} .

3 Theoretical formulation

3.1 Simplified momentum conservation equation (MCE)

The mean-flow equations solved in this work follow from Reynolds-averaging the Navier-Stokes equations, modelling the
 40 Reynolds stresses using the Boussinesq eddy-viscosity (ν_t) hypothesis. The time-averaged streamwise MCE reads, with ρ_0 the constant density, k the turbulent kinetic energy, and neglecting molecular diffusion ($\nu \ll \nu_t$):

$$u_j \frac{\partial u}{\partial x_j} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} - \frac{2}{3} \frac{\partial k}{\partial x} + \frac{\partial}{\partial x_j} \left(\nu_t \left(\frac{\partial u}{\partial x_j} + \frac{\partial u_j}{\partial x} \right) \right). \quad (1)$$



The turbine forcing is incorporated into the pressure gradient as a sudden pressure drop at the rotor disk (discussed later in Section 3.2), so it is not shown here as a separate term. To identify the terms that can be neglected while retaining the dominant physics, we perform a budget analysis of Eq. 1 for the flow within the wind-farm. The terms are computed over an area with the width and height of $2D$, laterally centred within the domain and spanning the vertical range $0 < z/D < 2D$. At each x , we compute the average of the absolute value of the budget terms across the section.

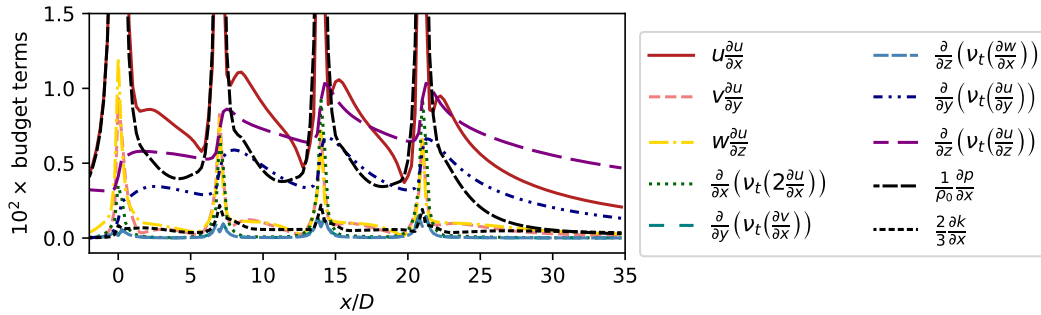


Figure 2. Streamwise evolution of the mean of the absolute values of the MCE budget terms, averaged at each streamwise location over a box of width and height $2D$ centred in the domain.

As shown in Fig. 2, streamwise advection dominates the advection terms, while the streamwise pressure gradient and lateral diffusion terms remains significant both near the rotor and in the downstream wake. The streamwise turbulent kinetic energy (TKE) gradient and axial diffusion are comparatively small and are therefore neglected. Next, we only keep the dominant terms in Eq. (1) and also express the pressure gradient using the approximated analytical solution that will be discussed later. This significantly simplifies Eq. (1) and enables solving the equation using streamwise marching. The final, simplified MCE considered in this work is given by:

$$u \frac{\partial u}{\partial x} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(\nu_t \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left(\nu_t \frac{\partial u}{\partial z} \right). \quad (2)$$

3.2 Pressure gradient

To compute pressure in Eq. (2), we employ the analytical solution recently proposed by Bastankhah et al. (2026). Under the assumption of small induction factors, the pressure Poisson equation reduces to Laplace's equation, $\nabla^2 P \approx 0$. Laplace's equation is solved separately for the upstream and downstream regions of each rotor on a semi-infinite domain. The pressure immediately upstream ($P_D^+ = \frac{1}{2} \rho_0 u_{0,i}^2 a(2-a)$) is used as a boundary condition for the upstream region, and the pressure immediately downstream ($P_D^- = P_D^+ - \frac{1}{2} \rho_0 u_{0,i}^2 C_T$) is used as a boundary condition for the downstream region, where a is the induction factor, C_T is the thrust coefficient, and $u_{0,i}$ is the inflow velocity. For rotor disk i , the pressure field is given by

$$\frac{p_i(x, r)}{p_{\text{ref}, i}} = \int_0^R \frac{2X_i r' E \left(\frac{2\sqrt{r' r_i}}{\sqrt{X_i^2 + (r_i + r')^2}} \right)}{\pi \sqrt{X_i^2 + (r_i + r')^2} (X_i^2 + (r_i - r')^2)} dr', \quad (3)$$



where X_i denotes the streamwise distance from rotor disk i , r_i is the radial distance from the centre of the i -th disk, E is the complete elliptic integral of the second kind, and R is the disk radius. For the upstream region (i.e. $X_i < 0$), the reference pressure is $p_{\text{ref},i} = P_D^+$, and for the downstream region (i.e. $X_i > 0$), $p_{\text{ref},i} = P_D^-$.

3.3 Turbulence closure: eddy viscosity ν_t

Classical linear eddy-viscosity closures yields a model that is often too dissipative in a wind turbine near-wake region, where the velocity gradients are high (van der Laan et al., 2015). To address this, we adopt the so-called $k - \varepsilon - f_p$ model introducing a flow-dependent scalar function f_p (van der Laan et al., 2015): $\nu_t = C_\mu f_p k^2 / \varepsilon$, with C_μ a constant. The function f_p reads:

$$f_p = \frac{2f_0}{1 + \sqrt{1 + 4f_0(f_0 - 1)\left(\frac{\sigma}{\tilde{\sigma}}\right)^2}}, \quad (4)$$

with $f_0 = C_R / (C_R - 1)$ and C_R the Rotta constant, $\tilde{\sigma} = 1 / \sqrt{C_\mu}$, and $\sigma = k / \varepsilon \sqrt{(\partial u_i / \partial x_j)^2} \approx k / \varepsilon \sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2}$ the shear parameter. Following similar simplifications as for the MCE, see also Blondel et al. (2026), the transport equations for k and ε (Launder and Spalding, 1974) are given by:

$$u \frac{\partial k}{\partial x} = \nu_t \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right] + \left[\frac{\partial}{\partial y} \left(\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial z} \right) \right] - \varepsilon, \quad (5)$$

$$u \frac{\partial \varepsilon}{\partial x} = \nu_t C_{1\varepsilon} \frac{\varepsilon}{k} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right] + \left[\frac{\partial}{\partial y} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial y} \right) + \frac{\partial}{\partial z} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) \right] - C_{2\varepsilon} \frac{\varepsilon^2}{k}. \quad (6)$$

The set of constants suggested in van der Laan et al. (2015), provided in table 1, closes the system.

C_R	C_μ	$C_{\varepsilon,1}$	$C_{\varepsilon,2}$	σ_k	σ_ε	κ
4.5	0.03	1.21	1.92	1.00	1.30	0.41

Table 1. $k - \varepsilon - f_p$ model constants, default values from van der Laan et al. (2015).

3.4 Numerical solution

Eqs. 2, 5, and 6 are discretised using a marching forward scheme in the x -direction based on an alternating direction implicit scheme, a common choice for such equation system due to its computational performance and strong stability (see, e.g., Bradstock and Schlez (2020)). The inflow conditions are based on the precursor simulations introduced in section 2. At the ground, a rough wall condition is imposed, following Hargreaves and Wright (2007), similarly to the RANS simulation. Neumann conditions are assumed at the domain top and on the sides. Following Calaf et al. (2010), we estimate each rotor's reference velocity as $u_{0,i} = u_{\text{disk},i} / (1 - a)$, updating the disk-averaged velocities from successive flow solutions until their maximum change falls below 10^{-4} , with initial values $u_{\text{disk},i} = u_h(1 - a)$. This process typically converges within five iterations.



85 4 Comparison to RANS simulations

The proposed methodology is verified against the reference RANS simulations using the same grid. We first consider a single turbine with four inflow turbulence intensities and two thrust coefficients. Figure 3 shows the centreline velocity deficits. Overall, very good agreement is obtained between the RANS and FM-RANS simulations across the tested inflow and turbine

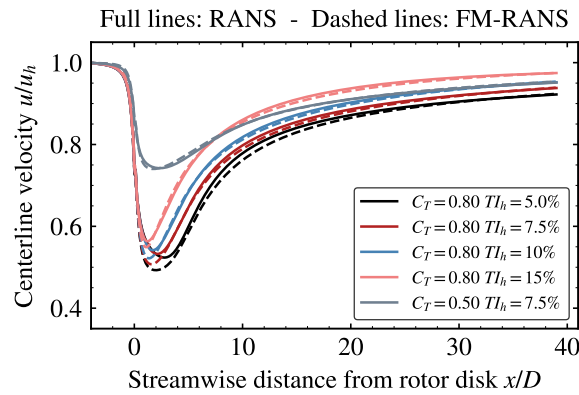


Figure 3. Comparison of the normalised centreline velocity along the wind turbine wake using the RANS and the FM-RANS approach.

operating conditions. Upstream of the turbine, the close agreement indicates that the analytical pressure model adequately captures the induction zone. Downstream, the velocity deficit is slightly overestimated, particularly for the lower inflow turbulence intensity. Beyond approximately $3D$, however, both approaches predict nearly identical wake recovery.

We also compare predictions for a column of four turbines, with a hub-height turbulence intensity of $I_{k,h} = 7.5\%$. Figure 4 shows contours of streamwise velocity u , turbulent kinetic energy k , its dissipation rate ε , and streamwise pressure gradient $\partial p/\partial x$ between the FM-RANS and RANS approaches. Similar to the single-turbine case, there is a slight discrepancy in near-wake quantities, where the FM-RANS tends to slightly overestimate the velocity and underestimate the TKE and its dissipation. However, overall the contours demonstrate the ability of the FM-RANS solution to reproduce the RANS results, with very good agreement between FM-RANS and RANS methods (especially for u and k). Not shown here for brevity, but the error of FM-RANS in comparison with the standard RANS method to model the wake velocity deficit is less than 1.0%.

4.1 Computational costs

For the single turbine case, the FM-RANS simulation ran in roughly 30 s on a single core of a INTEL(R) CORE(TM) ULTRA 7 265H central processing unit (CPU), converging within four iterations. Similar RANS simulation took roughly 4 h on 8 CPU cores. The comparison of computational costs between the two methods is rather similar for the wind farm cases: the FM-RANS solution ran in roughly 50 s and converged within five iterations, while the RANS solution took roughly 4 h on 8 CPU cores. The per-core-normalised speedup are of the order of approximately 2000 to 7500 $\times$ depending on the test case.

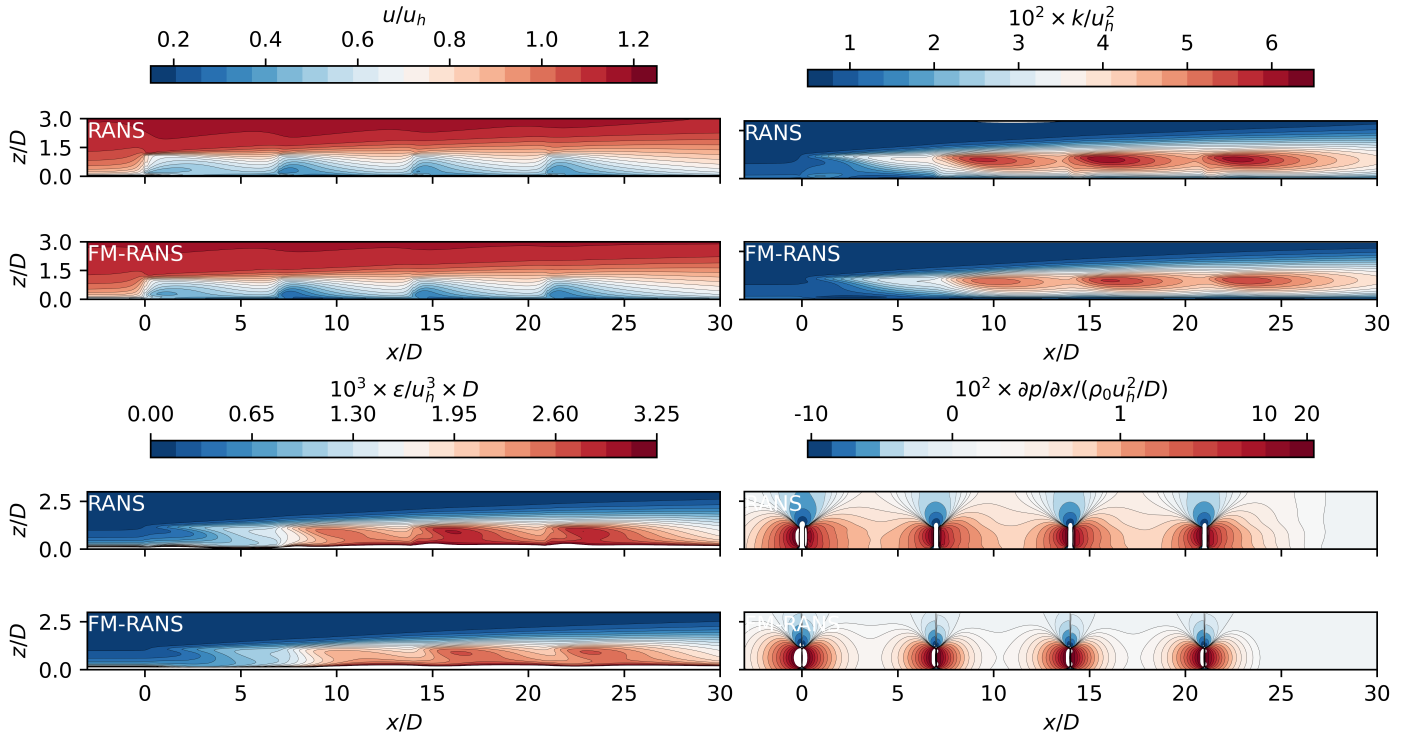


Figure 4. Contour plots of the streamwise velocity (upper left), TKE (upper right), TKE dissipation rate (lower left), and streamwise pressure gradient (lower right) on constant- y planes centred at the hub location.

105 5 Conclusions

A parabolic RANS framework, FM-RANS, coupled with an analytical model for the pressure gradient, has been introduced. Combined with the $k - \epsilon - f_p$ non-linear eddy-viscosity model, the framework enables continuous induction-to-far-wake prediction in wind farm flows (with no need for near-wake corrections) at a modest computational cost, on the order of one minute on a standard laptop processor. The predictions obtained with the new method are validated against full RANS equations, with
 110 our simplified approach costing only a fraction of full RANS. Several avenues remain for further development. In particular, the physical modelling could also be extended to account for veer, atmospheric stability and complex terrain.

Author contributions. All authors: proposed the methodology, conducted formal analysis, and carried out the investigation. FB: implemented the model, ran the simulations, and wrote the manuscript draft. MB: reviewed and edited the paper.

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 115 competing interests.



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