



Validation of RANS-calibrated engineering models and ANN-based surrogate for wind farm flow simulation and layout optimization

Jens Peter Schøler¹, Ernestas Simutis¹, M. Paul van der Laan¹, Julian Quick¹, and Pierre-Elouan Réthoré¹

¹DTU Wind and Energy Systems, Frederiksborgvej 399, 4000 Roskilde, Denmark

Correspondence: Jens Peter Schøler (jpsch@dtu.dk) and Pierre-Elouan Réthoré (pire@dtu.dk)

Abstract. Accurate yet efficient wake modeling is essential for wind farm layout optimization (WFLO). Wind turbine wakes are disturbed regions of flow behind a wind turbine, characterized by lower mean wind speeds and higher turbulence, which reduce downstream power production and increase structural loading. This study compares an Artificial Neural Network (ANN)-based surrogate trained on Reynolds Averaged Navier-Stokes (RANS) data with two representative engineering wake models based on the TurbOPark and Super-Gaussian formulations. The work includes recalibration of the engineering models, a systematic flow simulation study across varying turbine counts and spacings, and WFLO benchmarks validated against RANS-based Annual Energy Production (AEP). Results show that the ANN surrogate achieves the lowest RMSE and MAPE across all scenarios in flow estimation, albeit at a higher computational cost. In WFLO, the TurbOPark-based model produced the highest RANS-validated AEP layouts, despite having lower predictive accuracy, suggesting that optimization complexity influences outcomes. Blockage modeling increased computational cost without improving accuracy.



1 Introduction

A key challenge when designing a wind farm is determining the optimal placement of wind turbines. This decision must consider various factors, including nearby communities, local wind conditions, electrical infrastructure, seabed conditions, and other relevant considerations. The interaction between turbines through wake effects is particularly important, as it can significantly reduce energy output and, in turn, impact project economic viability.

To obtain the best achievable placement of the turbines Wind Farm Layout Optimization (WFLO) can be applied. WFLO utilizes numerical optimization to minimize or maximize a given cost function, typically a variation of energy production or energy cost, while adhering to predefined constraints (see e.g., Herbert-Acero et al., 2014; Baker et al., 2019; Bortolotti et al., 2022). In WFLO, it is essential for flow models to account for wake effects. Existing approaches can be broadly classified into two categories: engineering wake models, which construct the wind farm flow field by superposing individual wakes, and Computational Fluid Dynamics (CFD) models, which resolve the underlying physics through numerical simulation.

The first engineering model, proposed by Jensen (1983) and often referred to as the Park model, was primarily concerned with representing the wake-centre velocity and employed a top-hat shape to do so. This approximation initially worked well, but became increasingly problematic as the turbine and wind farm sizes increased. The breakdown of turbulent structures leads to the wake becoming self-similar and converging toward a Gaussian-like shape. For this reason, later variations typically rely on a Gaussian profile to model the wake deficits. The area where the wake is self-similar is referred to as the far wake. The location downstream of the turbine where this occurs depends on both atmospheric and terrain conditions. Dar et al. (2019) investigated this with LES simulations and showed that self-similarity occurs 1 to 3 rotor diameters (D) in complex terrain and between 3 to 10 rotor diameters for flat terrain. The different areas of the flow around a wind turbine are labeled and illustrated in Fig. 1.

In the literature, engineering models and CFD methods are often referred to as low- and high-fidelity models, respectively. As implied, the low-fidelity engineering models are not considered as accurate as high-fidelity CFD models; however, they are much faster. Compared to Reynolds Averaged Navier Stokes (RANS), the engineering models are $\sim 10^4 \times$ faster (van der Laan et al., 2015b) and compared to Large Eddy Simulation (LES) $\sim 10^7 \times$ (Porté-Agel et al., 2020). The CFD methods are too slow for an application like WFLO, where the flow needs to be evaluated multiple times during the optimization process. At the same time, the accuracy of the engineering models is challenged by optimization algorithms that, by nature, exploit and/or avoid areas of low accuracy. To mitigate the most adverse effects of this, a minimum constraint is often imposed on the inter-turbine separation, as engineering models are tuned to the self-similar far wake and generally perform poorly in the near wake and early transition zone.

40

An alternative to classical models is the use of data-driven surrogate models. Surrogates are models that seek to approximate a higher-fidelity model at a fraction of the cost. In wake modeling, surrogates are typically trained with RANS or LES data. Zehtabiyani-Rezaie et al. (2022) has conducted a review of the field, encompassing various types of Artificial Neural Network (ANN) based wake surrogates, as well as classic Machine Learning (ML) techniques such as Proper Orthogonal Decomposi-

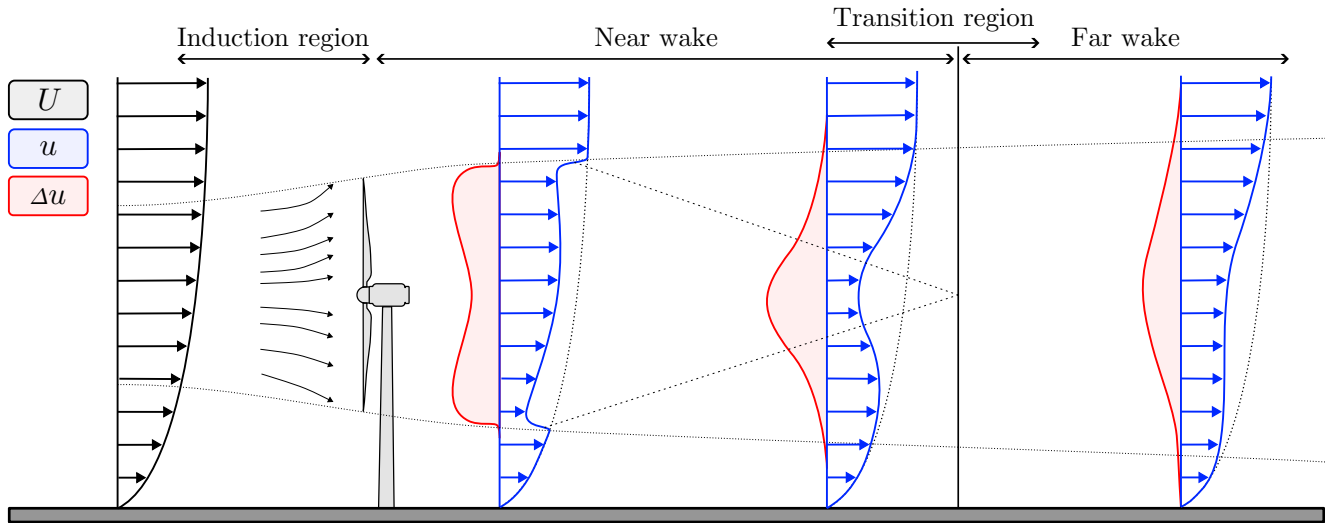


Figure 1. Simplified illustration of the flow around a wind turbine, depicting the upstream induction zone and the downstream wake, which is separated into the near wake, transition region, and far wake. The illustration includes the effective wind speed and the velocity deficit, illustrating the influence of the turbine on the background flow.

tion (POD) and Dynamic Mode Decomposition (DMD). Some notable works include Ti et al. (2021); Yang and Deng (2023), who trained an ANN-based RANS surrogate using multiple parallel neural networks, each corresponding to a specific grid location in the turbine flow. A limitation of this approach is that the surrogate cannot be used with automatic differentiation in gradient-based WFLO or any optimization with continuous turbine coordinates, as these models are undefined outside the considered grid points. To address this, we have previously developed ANN-based RANS surrogates with continuous definitions that support automatic differentiation (Schøler et al., 2023; Pish et al., 2024). One limitation of training ANN-based surrogates is the need for extensive data to cover cases across various inflow types and operating conditions. Schøler et al. (2024) investigated Physics-Informed Neural Networks (PINNs) as a means to reduce data requirements, ultimately finding that a hybrid data-and-physics loss was too costly given the gains. Gafoor et al. (2025) demonstrated the possibility of an RANS PINN surrogate trained solely with physics information. Together, current literature indicates a significant potential in ANN-based RANS surrogates.

While numerous studies on ANN wake surrogates hypothesize that these models can outperform traditional engineering approaches in flow simulation and WFLO accuracy, numeric validation studies remain limited. Anagnostopoulos and Piggott (2022) trained an ANN surrogate using data from an engineering model and applied it to both yaw optimization for a fifteen-turbine array and layout optimization for a six-turbine configuration. Similarly, Sun and Yang (2023) developed an ANN surrogate trained on data from an engineering model and conducted a WFLO study incorporating hub-height optimization for a thirty-turbine wind farm. Yang et al. (2022) and Yang and Deng (2023) demonstrated the ANN-RANS surrogates developed



by Ti et al. (2021) in yaw optimization of a five-turbine row and in a WFLO re-powering study of the Horns Rev 1 offshore wind farm, respectively. However, neither study validated their results against CFD simulations or against Supervisory Control and Data Acquisition (SCADA) data. In contrast, Li et al. (2024) introduced a novel RANS-based ANN surrogate trained using a Generative Adversarial Network (GAN). Their study included flow simulation via the superposition of single-wake models applied to the Horns Rev 1 wind farm, with power production results compared against RANS, time-averaged LES, and SCADA data, though only for a limited range of inflow cases. Likewise, Liu et al. (2025) trained a Convolutional Neural Network (CNN) surrogate using time-averaged LES data and validated it using time-averaged LES simulations from a row of five turbines with varying inter-turbine spacings.

In summary, the existing validation studies in the literature have been limited by surrogate training and validation using low-fidelity engineering model data, WFLO without validation, flow studies with few turbines or limited variation in the studied farm layouts, and restricted inflow conditions. In this work, systematic experiments are conducted to validate a pre-developed RANS-ANN for flow simulation and WFLO. The contributions of this work include the following two main items, a flow study and a WFLO study:

- 1a. Re-calibration of engineering models to a RANS-based single wake dataset.
- 1b. Comparative study investigating the accuracy of an ANN wake model and two popular engineering model setups for wind farm flow simulation with a varying amount of wind turbines, including a cost-benefit analysis of accuracy, memory consumption, and computational cost.
2. WFLO validation study of the ANN wake model against two engineering model setups and three different optimization algorithms. Validation of the optimized layouts with RANS-based Annual Energy Production (AEP) estimates.

The paper consists of Sect. 2, which outlines the studies conducted and the methods used. In Sect. 3, the results of the studies are presented and discussed. In Sect. 4, the conclusions of the study are summarized, and suggestions for future work are presented.

2 Methodology

In this section, the methods used in the paper are presented, and the experiments conducted are described.

2.1 Reynolds Averaged Navier-Stokes (RANS)

RANS is a steady-state CFD model that solves the mean flow by modeling all turbulence scales. In this work, the effects of turbulence are modeled with the $k\text{-}\varepsilon\text{-}f_P$ eddy viscosity model (van der Laan et al., 2015a, b). The inflow represents a logarithmic profile, where the ambient turbulence intensity (TI) based on the turbulent kinetic energy at hub height, is set by the roughness length. For the WFLO AEP study, the TI is set to 6%, yielding an offshore roughness length of 2.5×10^{-4} m. The turbines are modeled by Actuator Disks (AD) Troldborg et al. (2015). The AD thrust and tangential force distributions utilize

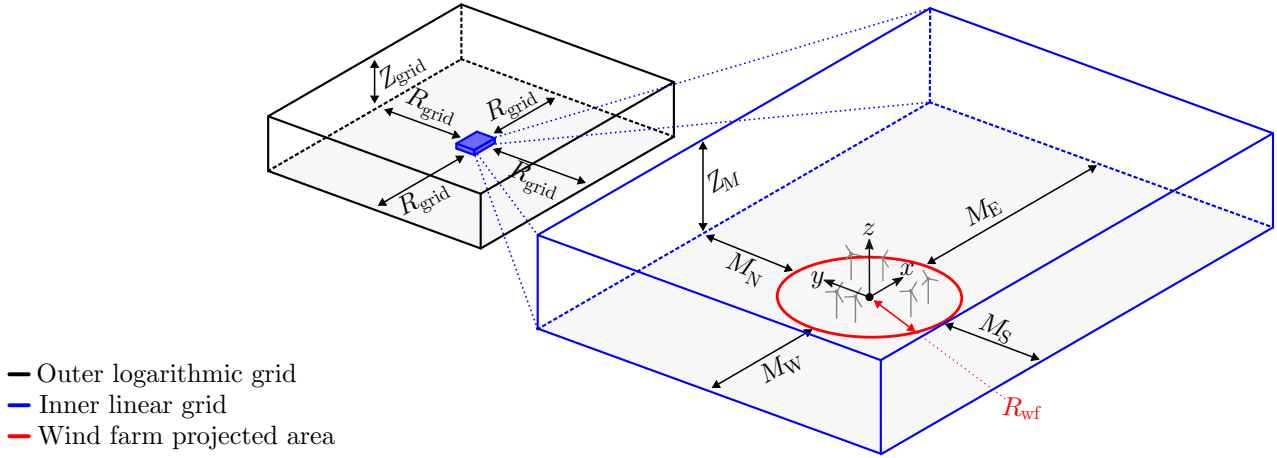


Figure 2. Illustration of utilized PyWakeEllipsys-flatbox mesh.

an analytical Joukowsky rotor model, as proposed by Sørensen et al. (2020), but recalibrated by van der Laan et al. (2020) for improved performance under veer and shear. The RANS simulations are performed with the PyWakeEllipsys code (DTU
 95 Wind and Energy Systems, 2023), which wraps the EllipSys3D finite volume flow solver (Michelsen, 1992; Sørensen, 1994). A Cartesian mesh with inner and outer grids was used, as illustrated in Fig. 2, where generic parameters are introduced; these are described in Tab. 1, which also includes the specific values considered in this study.

Table 1. Parameters for PyWakeEllipSys-flatbox mesh considered during simulations.

Parameter	Description	Considered values
Z_{grid}	Outer flow domain height	$25D$
R_{grid}	Outer flow domain extend	$500D$
M_N	Northern inner domain length	$3D$
M_E	Eastern inner domain length	$20D$
M_S	Southern inner domain length	$3D$
M_W	Western inner domain length	$5D$
Z_M	Inner box height	$3D$
Δs	Grid spacing	$D/8$

The WFLO RANS AEP simulations are performed for all wind directions and wind speeds with intervals of 5° and 1 ms^{-1} , respectively, yielding 1584 inflow cases. The simulations are run consecutively, with a new wind direction simulated by rotating
 100 the wind farm layout and a new wind speed by scaling the turbine controllers according to Reynolds number similarity (van der Laan et al., 2019), while the inflow is kept constant. This reduces the total number of required iterations because only local changes need to be recalculated, as discussed by van der Laan et al. (2022). Furthermore, the wind speed cases are simulated



from low to high, and the wind speed cases above the wind farm rated wind speed are skipped, which reduce the total number of flow cases by about 40%. Finally, a relative low convergence level of 10^{-4} is applied, which reduces the computational effort by an order of magnitude, while the convergence error in terms of AEP is 0.02%, as shown in van der Laan et al. (2022). A single AEP simulation with this RANS setup takes between 28 and 33 hours employing 1000 cores using the Sophia High Performance Computer (HPC) (Technical University of Denmark, 2019a), which is equipped with first generation AMD EPYC 7351 cores (released in 2017). Tests of PyWakeEllipSys RANS simulations on a different HPC with more modern fourth generation AMD Genoa-X cores (released in 2023) shows a speed up of a factor 2.

2.2 Engineering models

In this work, the PyWake engineering model framework (Pedersen et al., 2023) was used as the engine to run wind farm simulations. PyWake is a highly efficient framework that provides a high degree of abstraction for the components commonly found in low-fidelity wind farm models. The vast number of module combinations means only a subset can be considered at a time. We have chosen to consider a configuration based on the Turbulence Optimized Park model (TurbOPark) (Nygaard et al., 2020, 2022). The configuration is altered slightly and is therefore generally referred to as the TP model to avoid confusion with the original TurbOPark (Nygaard et al., 2022). The second alternative considered is based on a configuration submitted to the American WAKE Experiment (AWAKEN) (Bodini et al., 2026), which utilizes the super-Gaussian model proposed by Blondel and Cathelain (2020) and is therefore referred to as the SG model.

Regardless of the wake model used to represent the wind farm flow field, wake superposition is required. This is typically performed using either linear or quadratic methods. In the present work, linear superposition is applied for velocity deficits (Δu) in both configurations, while a quadratic max sum is used for added turbulence intensity (I_a) in the SG configuration. An overview of superposition methods is provided by Porté-Agel et al. (2020).

For readers already familiar with engineering wind farm models, a summary of the two configurations is shown in Tab. 2, while the models are presented in greater detail in the remainder of the subsection for previously unfamiliar readers.

Turbo Park (TP) model

The TP model uses the updated Turbulence Optimized Park model (TurbOPark) Nygaard et al. (2022), a Gaussian wake model derived from the classic Gaussian wake model by Bastankhah and Porté-Agel (2014). The wake model is shown in Eq. 1.

$$\frac{\Delta u(x)}{U} = C(x) \exp\left(-\frac{r^2}{2\sigma_w^2(x)}\right) \quad (1a)$$

$$C(x) = 1 - \sqrt{1 - \frac{C_T}{8(\sigma_w(x)/D)^2}} \quad (1b)$$

where $\Delta u(x)$ is the wake deficit at the downstream position x , U is the free-stream velocity, $C(x)$ is the peak velocity deficit at the wake centerline, r is the radial distance from the wake center, C_T the coefficient of thrust, and $\sigma_w(x)$ is the variable wake expansion factor, which is the major difference between the TurbOPark implementation and the default Gaussian deficit model which uses a constant wake expansion factor. The expression for the wake expansion factor $\sigma_w(x)$ is implemented using the



Table 2. Comparison of PyWake configurations (with/without blockage). Values flanked by dashes indicate shared settings across both blockage configurations. A single dash (–) indicates that the component is not used.

Component	SG		TP	
	w/o Blockage	w/ Blockage	w/o Blockage	w/ Blockage
Wake model	– Blondel and Cathelain (2020) –		– Nygaard et al. (2022) –	
Blockage model	–	Forsting et al. (2023)*	–	Forsting et al. (2023)*
Turbulence model	– Crespo and Hernández (1996) –		–	Crespo and Hernández (1996)
Superposition (Δu)	– Linear –		– Linear –**	
Superposition (I_a)	– Quad. Max. Sum –		–	Quad. Max. Sum
Wind farm model	Prop. Downwind	All2All Iter.	Prop. Downwind	All2All Iter.
Ground model	– Mirror –		– Mirror –	
Rotor averaging	– Center –		– Gaussian overlap –	

* Updated version of (Trolborg and Meyer Forsting, 2017).

** In the original TurbOPark paper, quadratic superposition was employed; however, this approach is incompatible with the blockage model (Forsting et al., 2023).

Frandsen (2005) turbulence model, which is why the TurbOPark model does not require a standalone added turbulence model.

135 $\sigma_w(x)$ is shown in Eq. 2 parameterized with α and β from Frandsen (2005).

$$\frac{\sigma_w(x)}{D} = \varepsilon + \frac{AI_0}{\beta} \cdot \left(\sqrt{(\alpha + \beta x/D)^2 + 1} - \sqrt{1 + \alpha^2} \right. \\ \left. - \ln \left[\frac{\left(\sqrt{(\alpha + \beta x/D)^2 + 1} + 1 \right) \alpha}{(\sqrt{1 + \alpha^2} + 1)(\alpha + \beta x/D)} \right] \right), \quad (2a)$$

$$\alpha := c_{I,1} I_0, \quad \beta := \frac{c_{I,2} I_0}{\sqrt{C_T}} \quad (2b)$$

$$140 \quad \varepsilon := c_\varepsilon \sqrt{\frac{1}{2} \frac{1 + \sqrt{1 - \hat{C}_T}}{\sqrt{1 - \hat{C}_T}}}, \quad (2c)$$

where $\hat{C}_T := \min(C_T, C_{T,\lim})$

Where I_0 is the ambient TI, A is a wake expansion calibration parameter, $c_{I,1}$ and $c_{I,2}$ are calibration parameters from the Frandsen (2005) model, and ε is an initial characteristic wake width i.e. $\sigma_w(0)/D = \varepsilon$. Here, it is important to note that the engineering wake models operate under an assumption that $I_k = I_u$, where $I_k = \sqrt{2/3 \cdot k}/U$ is the TI derived from turbulent kinetic energy and $I_u = \sigma_u/U$ is the streamwise TI based on the standard deviation (std) of the flow in the streamwise direction

145 σ_u . This differs from the more physically accurate relationship $I_k \approx 0.8 \cdot I_u$, derived from standard atmospheric turbulence ratios (Panofsky and Dutton, 1984). However, this simplification is not expected to alter the main conclusions, as the calibrated models exhibit similar overall trends.



Super Gaussian (SG) model

The SG model, as previously mentioned, uses the super-Gaussian model proposed by Blondel and Cathelain (2020); here, we present the simplified analytical version. The model unifies the observation that the far wake can be accurately modeled as a Gaussian, whereas the near wake more closely resembles a top-hat shape. By augmenting the super-Gaussian shape parameter n , the velocity deficit takes on different forms, i.e., when $n = 2$ it becomes a regular Gaussian shape, while for $n > 2$ the shape gradually becomes more top-hat-like.

$$\frac{\Delta u(x)}{U} = C(x) \exp\left(-\frac{(r/D)^n}{2(\sigma(x)/D)^2}\right) \quad (3a)$$

$$C(x) = 2^{2/n-1} - \sqrt{2^{4/n-2} - \frac{nC_T}{16\Gamma(2/n)(\sigma(x)/D)^{4/n}}} \quad (3b)$$

$$\frac{\sigma(x)}{D} = (a_s I + b_s)x/D + c_s \sqrt{\frac{1}{2} \frac{1 + \sqrt{1 - C_T}}{\sqrt{1 - C_T}}} \quad (3c)$$

$$n \approx a_f \exp(b_f \cdot x/D) + c_f, \quad a_f := 3.11 \quad (3d)$$

where Γ is the gamma function, I is the effective TI. Eq. 3d shows the exponential function that Blondel and Cathelain (2020) created to avoid having to solve for n and is the key change to enable fast evaluations of the flow. The effective TI is evaluated using the added TI (I_a) model proposed by Crespo and Hernández (1996) and the quadratic sum. The Crespo and Hernández (1996) model is shown in Eq. 4. To evaluate the axial induction (a_m), the method by Madsen et al. (2020) as implemented in PyWake is utilized, and a quadratic max summation is used for superposition.

$$I_a = 0.73 a_m^{0.8325} I_0^{-0.0325} (x/D)^{-0.32} \quad (4a)$$

$$a_m = 0.083 C_T^3 + 0.0586 C_T^2 + 0.2460 C_T \quad (4b)$$

$$I = \sqrt{I_0^2 + \max_j (I_{a,j})^2} \quad (4c)$$

where j is an index indicating a neighboring turbine.

Blockage

While wake effects are the primary drivers of flow interactions between turbines, turbine induction also plays a role. In PyWake, these effects are modeled using blockage models. When blockage is considered, the Self-Similarity Deficit model by Trolldborg and Meyer Forsting (2017), as updated by Forsting et al. (2023), is employed.

$$\frac{\Delta u_b}{U} = a_0(x, C_T) \nu(x) \operatorname{sech}^{c_\alpha} \left(c_\beta \frac{r}{r_{1/2}(x)} \right) \quad (5a)$$



$$\nu(x) = \left(1 + \frac{x/R}{\sqrt{1 + (x/R)^2}} \right) \quad (5b)$$

where ν is the centre-line induction, a_0 is the axial induction factor, R is the rotor radius and $r_{1/2}(x)$ is a linear induction zone half radius, defined as:

$$\frac{r_{1/2}(x)}{R} = c_\lambda \cdot (x/R) + c_\eta, \quad (5c)$$

$$a_0(x, C_T) = \frac{1}{2} \left(1 - \sqrt{1 - \gamma(x, C_T) \cdot C_T} \right), \quad (5d)$$

Where c_λ and c_η are tunable parameters introduced to account for the effects of wind-farm blockage. In the updated blockage model (Forsting et al., 2023), the $\gamma(x, C_T)$ function was updated to gradually shift from a far-field formulation to a near-field formulation. The said γ function is introduced below, with an abstraction of the transition function reported as a $\delta(x)$ function:

$$\delta(x) = \begin{cases} 1 & \text{for } x/R < -6 \\ \frac{|\nu(x) - \nu|}{\nu(-6) - \nu(-1)} & \text{for } -6 \leq x/R \leq -1 \\ 0 & \text{for } -1 < x/R \end{cases} \quad (5e)$$

$$\gamma(x, C_T) = \left\{ \delta(x) \cdot \left(-0.06489 \cdot \sin \left(\frac{C_T - 0.4911}{-0.1577} \right) + 1.116 \right) + (1 - \delta(x)) \cdot (C_T^3 a_{nf} + C_T^2 b_{nf} + C_T c_{nf} + d_{nf}) \right\} \quad (5f)$$

The near-field expressions is parameterized with a_{nf} , b_{nf} , c_{nf} and d_{nf} . For the far-field parameters, the original values are used as reported in Eq. 5f.

2.3 Recalibration of deficit models

To ensure that the deficit models considered are compared fairly, they have been recalibrated to a single-wake RANS dataset. To construct the dataset, a pre-existing RANS Lookup Table (LUT) is used, which consists of 3D fields of velocity deficit and wake-added TI for all combinations of $C_T \in [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.923]$ and $I_0 \in [5, 10, 20, 30] \%$. The inflow represents a logarithmic profile where the roughness length is used to set I_0 , as described in subsection 2.1. The single-wake RANS dataset is constructed by linearly interpolating/extrapolating the RANS-LUT to wind speeds $U \in [4, 5, 6, 7, 8, 9, 10]$ m/s and turbulence intensities $I_0 \in [5, 10, 15, 20, 25, 30, 35, 40] \%$, covering the modeling space. Similar RANS single wake databases have been used to construct surrogate models in previous work Criado Risco et al. (2023); van der Laan et al. (2026).



The Bayesian optimization framework proposed by Nogueira (2014) is used to minimize the L^2 norm between the hub-height velocity fields of the RANS data and the deficit models. For each flow case, an L^2 norm is calculated, and the mean of those is used as the optimization objective function. The deficit models are calibrated to account for velocities between 2 and 10 rotor diameters downstream of the source turbine and up to 2 rotor diameters in the cross-flow direction. The upstream deficit model is calibrated considering the region between 1 and 2 rotor diameters downstream of the source turbine and up to 2 rotor diameters away in the cross-flow direction. The optimization is initialized with 50 parameter configurations sampled from uniform distributions (detailed in Tab. 3). A set of parameters is denoted in vector form as ϕ_i and a collection of them in matrix notation as Φ . A given set of parameters constitutes a realization of the wake deficit parameters shown in Tab. 3; the original uncalibrated parameters are not shown here but are included inside the results section with the optimized parameters in Tab. 6.

This implementation of Bayesian optimization uses Gaussian Process (GP) regression, here, the posterior predictive distribution for a test input ϕ_* is given by:

$$f(\phi_*) \mid \Phi, \mathbf{y} \sim \mathcal{N}(\mu_{\text{GP}}(\phi_*), \sigma_{\text{GP}}^2(\phi_*)) \quad (6a)$$

$$\mu_{\text{GP}}(\phi_*) = \mathbf{k}_*^\top (\mathbf{K} + \sigma_n^2 \mathbb{I}_n)^{-1} \mathbf{y} \quad (6b)$$

$$\sigma_{\text{GP}}^2(\phi_*) = k(\phi_*, \phi_*) - \mathbf{k}_*^\top (\mathbf{K} + \sigma_n^2 \mathbb{I}_n)^{-1} \mathbf{k}_* \quad (6c)$$

where $\mathcal{N}(\mu, \sigma^2)$ denotes a Gaussian distribution with mean μ and std σ , $\mathbf{y}$ is the vector of observed outputs at the n training points, σ_n can be used to account for observation and model uncertainty but here is just included for numerical stability with $\sigma_n = 10^{-3}$, $\mathbb{I}_n$ is the $n \times n$ identity matrix, and μ_{GP} and σ_{GP} are the mean and std obtained through GP regression. The covariance matrix $\mathbf{K}$ and test covariance vector $\mathbf{k}_*$ are constructed using a pre-specified kernel function. This framework uses a Matérn kernel for the GP regressor with smoothness parameter $\nu_M = 2.5$ and length scale set to 1, which can therefore be removed as it only occurs in denominators; the Matérn kernel is shown in Eq. 7.

$$k(\phi_i, \phi_j) = \frac{1}{\Gamma(\nu_M) 2^{\nu_M-1}} (\sqrt{2\nu_M} \|\phi_i - \phi_j\|_2)^{\nu_M} K_{\nu_M}(\sqrt{2\nu_M} \|\phi_i - \phi_j\|_2) \quad (7)$$

where $k(\phi_i, \phi_j)$ is the Matérn covariance between parameter sets ϕ_i and ϕ_j , K_{ν_M} is the modified Bessel function of the second kind, and Γ is the gamma function. The elements of the covariance matrix $\mathbf{K}$ are given by $K_{ij} = k(\phi_i, \phi_j)$, and the test vector is:

$$\mathbf{k}_* = \begin{bmatrix} k(\phi_1, \phi_*) & k(\phi_2, \phi_*) & \cdots & k(\phi_n, \phi_*) \end{bmatrix}^\top \quad (8)$$

To guide the selection of the next evaluation point, an acquisition function is required. This implementation uses the Upper Confidence Bound (UCB) acquisition function to decide the next best set of parameters. In this implementation an exploration parameter of $\kappa = 2.576$ is used for UCB as shown in Eq. 9.

$$\text{UCB}(\phi_*) = g(f(\phi_*) \mid \Phi, \mathbf{y}) = \mu_{\text{GP}}(\phi_*) + \kappa \sigma_{\text{GP}}(\phi_*) \quad (9)$$



The Bayesian optimization is run for 200 sequential iterations. In each iteration, the GP model is fitted to all previously observed parameter sets, the UCB acquisition function is maximized to select the following parameter set ϕ_* to evaluate, and the objective function is computed at this point. After all iterations, the parameters associated with the lowest error are reported.

Table 3. Parameter bounds for the considered wake models during the calibration process.

SG (Blondel and Cathelain, 2020)								
	a_s	b_s	c_s	b_f	c_f			
Lower	0.001	0.001	0.001	-2	0.1			
Upper	0.5	0.01	0.5	1	5			
TP (Nygaard et al., 2022)								
	A	$c_{I,1}$	$c_{I,2}$	c_ϵ	$C_{\text{T},\text{lim}}$			
Lower	0.001	0.01	0.01	0.01	0.01			
Upper	0.5	5	5	3	1			
Blockage (Forsting et al., 2023)								
	c_α	c_β	c_λ	c_η	a_{nf}	b_{nf}	c_{nf}	d_{nf}
Lower	0.05	0.05	-2	-2	-3	-3	-3	-3
Upper	3	3	2	2	3	3	3	3

2.4 Artificial Neural Network (ANN) surrogate

In this work, a pre-trained RANS-based wake surrogate developed by Pish et al. (2024), based on the work of Schøler et al. (2023), is employed for wind farm simulations and WFLO. The surrogate is based on a conventional ANN architecture, commonly referred to as a deep neural network, a fully connected feed-forward neural network, or a Multi-Layer Perceptron (MLP). The model takes the Cartesian position in space (x, y, z) , the yaw offset from the inflow direction (γ) , C_T , and the TI as inputs. While TI is considered during our experiments, yaw misalignment is not. Not addressing yaw misalignment was a conscious decision to limit the scope of the paper as it would entail a significant increase in additional case to evaluate with RANS.

In Eq. 10 the MLP is defined as a set of equations. Eq. 10a shows the input $\mathbf{s}$, Eq. 10b hidden layer l , and Eq. 10c shows the output of the MLP $f_\theta(\mathbf{s})$.

$$h^{(0)} = \mathbf{s} = [x, y, z, \gamma, C_T, I]^\top \quad (10a)$$

$$h^{(l)} = \sigma^{(l)} \left(W^{(l)} h^{(l-1)} + b^{(l)} \right), \quad l = 1, 2, \dots, L-1 \quad (10b)$$

$$f_\theta(\mathbf{s}) = W^{(L)} h^{(L-1)} + b^{(L)} \quad (10c)$$

Where $h^{(l)}$, $\sigma^{(l)}$, $W^{(l)}$, $b^{(l)}$ are the activation, activation function, weights, and bias at layer l , respectively, and L is the total number of layers. The model is trained with a single wake RANS dataset of the DTU10MW reference wind turbine (Bak et al.,



250 2013). The architectures of the employed models are summarized in Tab. 4 and a brief overview of the engineering model configuration for the ANN is presented.

Table 4. Summary of pre-trained RANS ANN surrogates, and model configuration details for engineering model type simulation. Values flanked by dashes indicate shared settings across both flow model outputs.

Parameter	Δu model	I_a model
x -range	[$-3D$, $70D$]	
y -range	[$-6D$, $6D$]	
Neurons pr. layer	(70, 102, 102, 102)	(118, 118, 118, 118)
Activation (σ)	(tanh, $3 \times$ sigmoid)	($4 \times$ sigmoid)
No. of params	28847	43071
ANN engineering model configuration		
Superposition (Δu)	– Linear –	
Superposition (I_a)	– Linear –	
Wind farm model	– Propagate Downwind w/o blockage / All2All Iterative w/ blockage –	
Ground model	– Mirror –	
Rotor averaging	– Center –	

2.5 Wind farm flow study

To evaluate the accuracy of the three low-fidelity options: the two engineering models and the ANN surrogate, we conduct a systematic study examining the influence of turbine count and inter-turbine spacing. Because flow construction based on
 255 superposition inherently introduces errors that depend on these parameters, varying them enables a structured investigation of construction error and facilitates a direct comparison of the engineering models and the ANN. Additionally, the study examines how these factors impact memory requirements and computational costs.

To examine the effect of turbine count, six base layouts are considered with $n_{wt} \in \{6, 10, 20, 30, 40, 50\}$. Fig. 3 illustrates these configurations. To assess the impact of inter-turbine spacing, each base layout is scaled by a separation factor $s_{wf} \in$
 260 $\{3D, 5D, 7D\}$, resulting in a total of eighteen distinct layouts. Each layout is simulated using RANS, the calibrated and uncalibrated engineering models, and the ANN surrogate. By comparing these results, trends in model performance can be identified and the effectiveness of the ANN surrogate evaluated.

In the flow study, each possible model-layout combination is evaluated under different inflow conditions consisting of wind speeds $U = 8.0 \text{ ms}^{-1}$ and 14 ms^{-1} , ambient turbulence intensities $I = 5\%$ and 10% , and wind direction angles θ ranging from
 265 270° to 315° in increments of 3° .

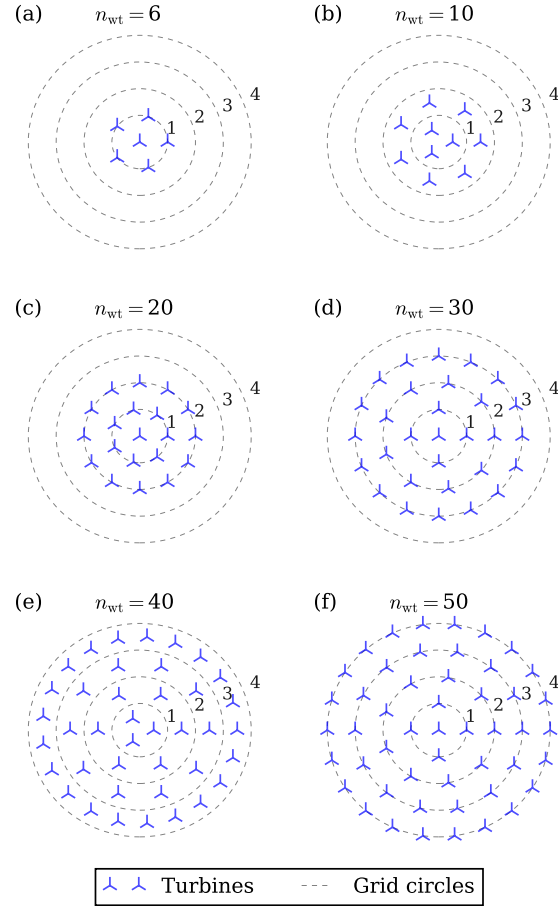


Figure 3. Circular turbine layouts evaluated in the wind farm flow study. The grid circles represent normalized radial positions, which are scaled by a separation factor $s_{wf} \in \{3D, 5D, 7D\}$ to generate configurations with varying inter-turbine spacing.

2.6 Wind farm layout optimization (WFLO)

To compare the low-fidelity models in WFLO, optimization is performed using each of the available low-fidelity models and the results are validated by estimating the Annual Energy Production (AEP) with RANS simulation. A WFLO case is formulated with the objective of maximizing AEP, subject to a minimum turbine separation $s_{\min} = 2D$ and polygonal boundary $\mathcal{P}$:



$$\begin{aligned}
 & \max_{\{(x_i, y_i)\}_{i=1}^{n_{wt}}} \text{AEP}(\{(x_i, y_i)\}_{i=1}^{n_{wt}}) \\
 & \approx 8760 \sum_{i=1}^{n_{wt}} \sum_{j=1}^{n_{\mathcal{U}}} \sum_{k=1}^{n_{\theta}} P_i(\{(x_m, y_m)\}_{m=1}^{n_{wt}}, \mathcal{U}_j, \theta_k) \rho(\mathcal{U}_j, \theta_k) \\
 & \text{s.t. } \sqrt{(x_i - x_q)^2 + (y_i - y_q)^2} \geq s_{\min}, \quad \forall i \neq q \\
 & (x_i, y_i) \in \mathcal{P}, \quad \forall i = 1, \dots, n_{wt}
 \end{aligned} \tag{11}$$

Where x_i , y_i , and P_i are coordinates and power produced at turbine i , and n_{wt} is the number of turbines. ρ is a probability mass function indicating the probability of seeing a given combination of wind speed bin $\mathcal{U}_j$ and binned wind direction θ_k . During optimization, the AEP is estimated with a wind direction bin for every 1° to avoid artificially unwaked areas which the optimization algorithms would exploit. However, during validation the bin discretization has been decreased to only consider a bin for every 5° to conserve computational resources driven by the cost of RANS simulations, as discussed in Sect. 2.1.

The optimization is performed with the DTU-developed software `Topfarm2` (Pedersen et al., 2025). Three different gradient-based optimization algorithms are considered as implemented in `Topfarm2`: Sequential Least Squares Quadratic Programming (SLSQP) (Kraft, 1988; Virtanen et al., 2020), Interior Point OPTimizer (IPOPT) (Wächter and Biegler, 2006), and Stochastic Gradient Descent (SGD) (Quick et al., 2023). In all cases, the gradients for the optimization are provided by the Automatic Differentiation included in `PyWake`.

Table 5. Parameters of considered optimization algorithms as used during studies with `TopFarm2`.

Parameter	SLSQP	IPOPT	SGD
Max. iterations	1,000	1,000	2,000
Convergence tolerance	1×10^{-3} MWh	1×10^{-2}	–
Initial learning rate	–	–	17.83 m
Momentum weight 1 (β_1)	–	–	0.1
Momentum weight 2 (β_1)	–	–	0.2
Final learning rate	–	–	0.01 m
Initial constant learning rate iterations	–	–	100
early stopping ratio	–	–	0.1

The considered site is based on the IEA 740-10-MW reference wind power plant (Kainz et al., 2024). The boundaries are altered to accommodate more turbines within a smaller projected wind farm area, as the RANS cost is proportional to the projected wind farm area. Figure 4 (a) shows the considered boundary conditions with the wind farm projected area, meaning the smallest circle that contains the whole farm. In Fig. 4 (b), the site flow conditions are depicted as a wind rose. As a simplification during optimization and validation, the ambient TI was set to $I_0 = 6\%$ for all flowcases. The layout optimization procedure is performed for each low-fidelity model, both with and without a blockage model, and for the engineering models

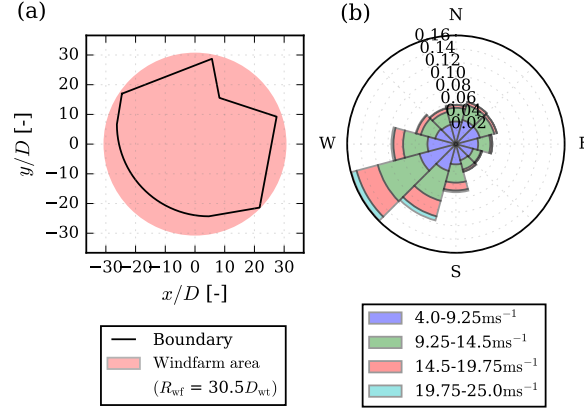


Figure 4. (a) Boundary constraint for the considered layout optimization (b) Wind rose at the considered IEA 740-10-MW reference site.

in their original and recalibrated forms, using multiple optimization algorithms and random initial layouts. For cases without
 290 randomly generated layouts to enable a consistent comparison across algorithms.

For cases including the blockage model, ten random initial layouts are considered; however, only the SGD algorithm is
 used, as it is less memory-intensive than the other approaches due to batch sampling of the wind rose. From the resulting set
 of optimized layouts, the layout yielding the highest AEP is selected for each low-fidelity model and subsequently validated
 using RANS simulations, thereby limiting the total number of RANS cases to one per low-fidelity model.

295 2.7 Evaluation metrics

To evaluate the relative performance of the different models, quantitative measures are needed. In this work, we use the Mean
 Absolute Percentage Error (MAPE), the Root Mean Square Error (RMSE), the maximum absolute error ($E_{\max}$), and the error
 standard deviation (σ_E).

$$\text{MAPE} = \frac{1}{N} \sum_{i=1}^N \left| \frac{u_i - u'_i}{U_i} \right| \cdot 100 \quad (12a)$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (u_i - u'_i)^2} \quad (12b)$$

$$E_{\max} = \max_i |u_i - u'_i| \quad (12c)$$

$$\sigma_E = \sqrt{\frac{1}{N-1} \sum_{i=1}^N \left((u_i - u'_i) - \frac{1}{N} \sum_{j=1}^N (u_j - u'_j) \right)^2} \quad (12d)$$



where u_i is the true wind speed for observation i , u'_i is the predicted wind speed, and N the total number of observations. In Eq. 12a the denominator uses the free-stream velocity U_i rather than u_i , in order to avoid numerical issues in heavily waked regions, where low wind speeds can lead to disproportionately large relative errors. Conversely, this formulation prevents large errors in unwaked regions from being underrepresented in the overall metric.

For flow field comparisons, regions within one rotor diameter of the turbines are omitted, making the analysis domain (Ω) a subset of the whole flow domain (Ω_{flow}). The whole flow domain is the hub-height flow plane, with grid parameters provided in Tab. 1 and shown in Fig. 2.

$$\Omega = \left\{ (x, y) \in \Omega_{\text{flow}} \mid \sqrt{(x - x_i)^2 + (y - y_i)^2} \geq D \quad \forall i = 1, 2, \dots, n_{\text{wt}} \right\} \quad (13)$$

Where this subset is used the metric will be marked with an Ω -subscript.

3 Results and Discussion

In this section, the results of the studies are presented and discussed.

3.1 Recalibration of wake model

The calibrated model parameters are summarised in Tab. 6. Overall, the changes are not dramatic: the sign of each parameter is preserved. However, several parameters do show substantial numerical differences.

Table 6. Original and recalibrated parameters for the considered wake- and blockage models.

SG (Blondel and Cathelain, 2020)					
	a_s	b_s	c_s	b_f	c_f
Org.	0.1700	0.0050	0.2000	-0.6800	2.4100
Recalib.	0.3332	0.0048	0.1625	-0.4242	2.9034
TP (Nygaard et al., 2022)					
	A	$c_{I,1}$	$c_{I,2}$	c_ϵ	$C_{T,\text{lim}}$
Org.	0.0400	1.5000	0.8000	0.2500	0.9990
Recalib.	0.2817	1.2016	3.1484	0.1421	0.7751
Blockage (Forsting et al., 2023)					
	c_α	c_β	c_λ	c_η	
Org.	0.8889	1.4142	-0.672	0.4897	
Recalib.	0.6208	2.6617	-0.5066	1.8932	
	a_{nf}	b_{nf}	c_{nf}	d_{nf}	
Org.	-1.381	2.627	-1.524	1.3360	
Recalib.	-2.1531	2.0948	-2.7612	2.9041	



For the SG model, most changes are relatively small, except for a_s , which increases from 0.17 to 0.33. This parameter governs the relationship between wake width and rotor thrust. Because the original calibration was based on a mix of experimental data, whereas the present work uses RANS data, different thrust and wake-width characteristics are expected; therefore, a noticeable change in a_s is expected.

For the TP model, the parameter $c_{I,2}$ changes markedly from 0.80 to approximately 3.15. The parameters $c_{I,1}$ and $c_{I,2}$ impact the auxiliary variables α and β in Nygaard et al. (2020) that together act as replacements for both the ambient TI and C_T in the wake expansion formulation. The initial values for $c_{I,1}$ and $c_{I,2}$ were lifted from (Frandsen, 2005; IEC, 2010). The original value was derived from measurement data, whereas the present RANS simulations employ a fundamentally different turbulence formulation. Consequently, a substantial adjustment is required for consistency with the RANS flow.

In the self-similarity deficit blockage model, the polynomial-fit parameters a_{nf} , b_{nf} , c_{nf} and d_{nf} also vary considerably. Because these coefficients describe a polynomial rather than a directly interpretable physical relationship, the practical implications of their changes are difficult to interpret. The parameters c_β and c_λ , however, are more interpretable and show meaningful shifts. The parameter c_λ represents an offset in the linear rotor half-radius induction zone, introduced to correct for wind-farm blockage effects. Effects that are absent in the single-turbine RANS dataset used here. A significant change is therefore expected. The scaling factor c_β , which modulates the influence of the rotor half-radius, is similarly affected.

In Tab. 7, error metrics are collected to compare results before and after recalibration. The error metrics in Tab. 7 show

Table 7. Metrics before and after recalibration for the two considered models.

SG (Blondel and Cathelain, 2020)				
	RMSE	$E_{\max}$	σ_E	MAPE
Org.	0.0354	0.2224	0.0347	0.3460
Recalib.	0.0205	0.1588	0.0194	0.2005
TP (Nygaard et al., 2022)				
	RMSE	$E_{\max}$	σ_E	MAPE
Org.	0.0716	0.2679	0.0681	0.6733
Recalib.	0.0167	0.2850	0.0165	0.1563
Blockage (Forsting et al., 2023)				
	RMSE	$E_{\max}$	σ_E	MAPE
Org.	0.0039	0.0131	0.0023	0.0500
Recalib.	0.0019	0.0079	0.0018	0.0201

improvements across all measures for the SG and blockage models. For the TP model, most metrics also improve, except for the maximum absolute error. This exception was surprising, but as the remaining metrics improved, it is expected that this was due to a large error near the rotor. This idea is supported by inspecting the spatial distribution of the mean RMSE (averaged over inflow conditions) in Fig. 5 (d), which indicates that the most significant errors occur immediately behind the turbine at $x = 2D$.



The overall takeaway from Fig. 5 is that the calibrated models yield substantially improved performance, particularly outside the near wake. The TP model exhibits the greatest overall improvement, which aligns with expectations: it was initially intended for farm-to-farm wake interactions and is therefore not expected to perform optimally in the intra-farm or single-wake context without recalibration.

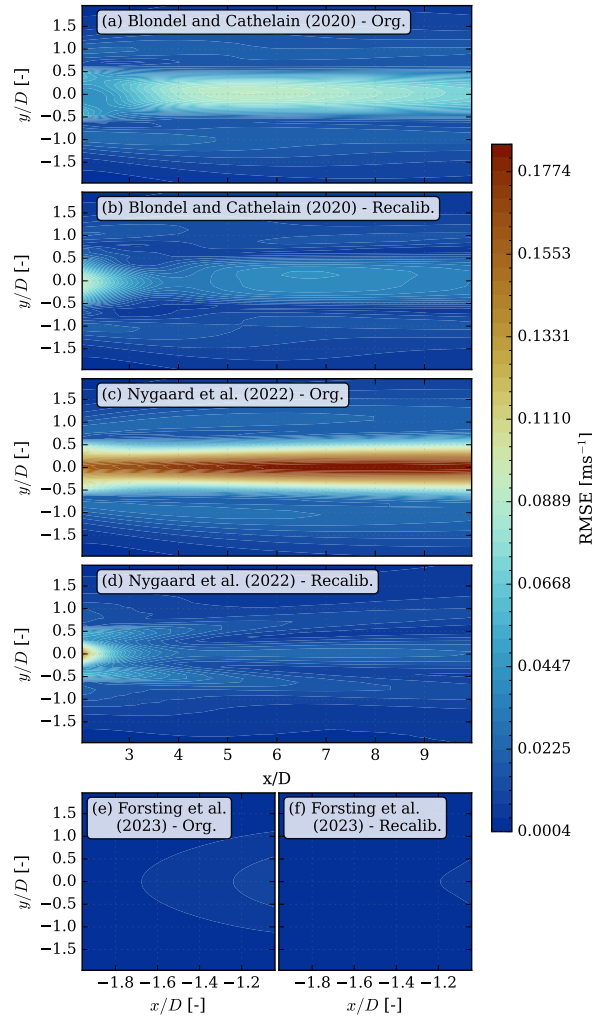


Figure 5. Wake- and blockage model calibration RMSE error maps

3.2 Wind farm flow study

The layouts introduced in Fig. 3 were evaluated with separation factors $s_{wf} \in \{3D, 5D, 7D\}$. Flow simulations were performed using all the available low-fidelity models (SG model, TP model, and the ANN), with the engineering models assessed in both calibrated and uncalibrated forms. To distinguish the calibrated and uncalibrated models a superscript (†) is used with



uncalibrated models e.g. $SG^{\dagger}$ refers to the uncalibrated SG model. Similarly, to distinguish between the models with and without blockage the superscript (*) is used to indicate the inclusion of blockage e.g. $SG^{\dagger*}$ indicates the uncalibrated SG model with blockage.

To quantify the accuracy of the lower-fidelity simulations, the RANS simulations described in subsection 2.1 were used as the reference ground truth. A masked flow domain, defined in Eq. 13, was applied to exclude the region immediately surrounding each turbine, as this region is irrelevant for optimization and does not represent a feasible turbine spacing. Fig. 6 presents $MAPE_{\Omega}$ and $RMSE_{\Omega}$ as functions of the number of turbines n_{wt} , with mean values indicated by solid lines and bootstrapped 95% confidence intervals shown as shaded regions. The ensemble mean is constructed from the considered inflow conditions, including wind directions, wind speeds, and ambient TIs.

Fig. 6 presents the results for the models without blockage; for completeness, the metrics with blockage are reported in Appendix A.

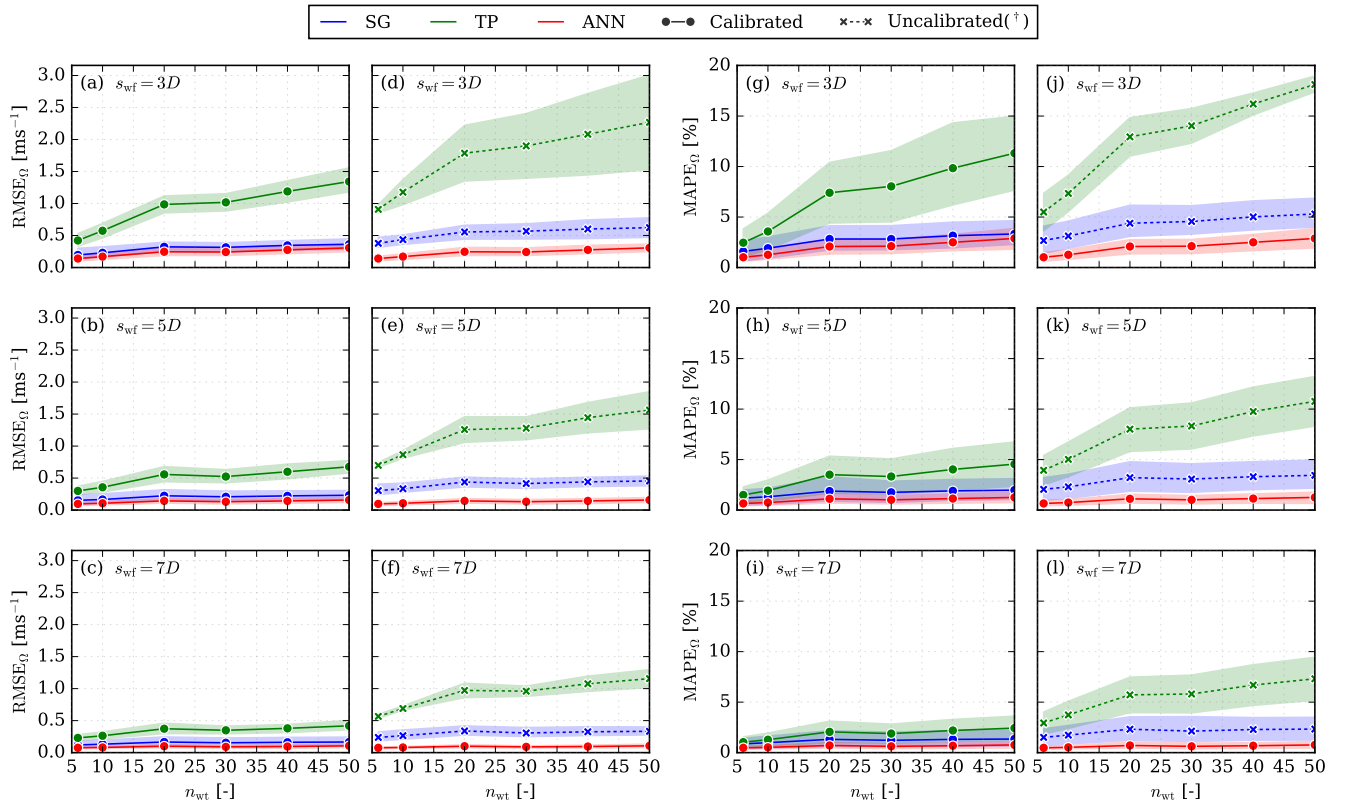


Figure 6. Flow study result metrics for an increasing number of wind turbines, without a blockage model. (a-c) $RMSE_{\Omega}$ with calibrated wake models, (d-f) $RMSE_{\Omega}$ with uncalibrated wake models, (g-i) $MAPE_{\Omega}$ with calibrated wake models, and (j-l) $MAPE_{\Omega}$ with uncalibrated wake models.



The results show that the error consistently decreases as the separation factor increases. This trend reflects the increasing complexity of the wake interactions at smaller inter-turbine distances. In these regimes, near-wake dynamics violate key assumptions in simplified wake models, particularly the superposition of individual wakes, which becomes progressively less valid as group effects strengthen (e.g., Andersen et al., 2014; Gunn et al., 2016). Simultaneously, the wake of a single turbine also behaves in a more complex manner in the near wake, further complicating the modeling of denser farms. Similarly, there is a shared trend of the error growing with the number of turbines, though at times it is subtle, which can be explained by the modeling error of superposing wakes, becoming increasingly problematic as the number of turbines increases. These trends between the error and the number of turbines and turbine density are observed for both the calibrated and uncalibrated engineering models. It is also observed that the SG model with the lowest recalibration error is the one with the lowest error here, increasing confidence in the recalibration process.

Furthermore, calibration improves the engineering models across both configurations, reducing the mean error and the width of the confidence intervals, indicating improvements in accuracy and robustness across wind directions and inflow conditions. An exception occurs for the $MAPE_{\Omega}$ of the TP model at $s_{wf} = 3D$, where calibration produces a lower mean error, but the confidence interval worsens. The reason is not immediately apparent, though it could be that the original model was intended for long-distance wakes and therefore produced consistent but poor results in the near wake, which would explain the poor performance.

Across all cases, the ANN-based surrogate outperforms the engineering models. Its superior performance is attributed to its ability to represent complex, spatially varying flow structures without relying on strong simplifying assumptions about wake behavior. This expressiveness enables the ANN to capture near-wake features with substantially higher fidelity, resulting in significantly lower errors than traditional engineering approaches.

In Fig. 7, the computational cost and memory consumption are plotted against the number of wind turbines. Figures 7 (a) and (b) both show the computational cost, but in Fig. 7 (a) the cost is expressed in CPU hours (CPUh), meaning that only models executed on the CPU can be compared directly. CPU hours are calculated by multiplying the walltime (in hours) by the number of CPU cores used during the simulation, allowing for a fair comparison. The nodes used have 32 cores, the Engineering models can be run on a single core, unlike the ANN, which by default uses all available cores. Figure 7 (b), on the other hand, reports the real walltime in seconds, enabling direct comparison with the GPU-accelerated ANN. All simulations have been conducted on identical and exclusive nodes on the DTU cluster Sophia (Technical University of Denmark, 2019b).

All the results are reported with a 95% confidence interval, based on an ensemble over the inflow conditions (wind direction, wind speed, and ambient TIs) and separation factors s_{wf} . The confidence is quite high, indicating that the variation in computational resource requirements with respect to layout density and inflow conditions is low. The results are shown on a semi-logarithmic plot, which likely contributes to the appearance of increasing confidence; however, it is also expected to be low.

Figure 7 (a) shows that, in terms of CPUh, the fastest model is the SG model, followed by the TP model, with the ANN being the slowest. The ANN being the slowest is expected, as evaluating it requires multiple matrix multiplications, as described in Eq. 10, whereas the engineering models require far fewer computations. The TP model employs a Gaussian overlap for the

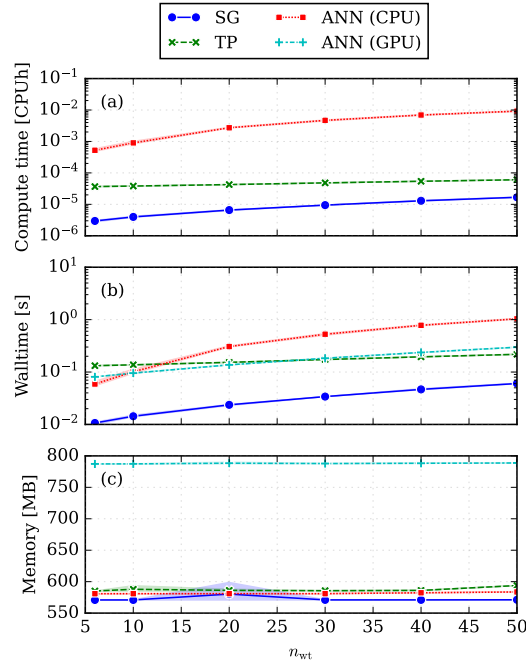


Figure 7. Average computational resource cost of a flow case for increasing wind-farm sizes without inclusion of blockage. (a) Relative cost in CPU hours, (b) real-world cost in walltime (seconds), and (c) Random Access Memory (RAM) consumption in MB.

rotor averaging strategy, which requires additional flow evaluation and appears to increase its computational cost; however, this relative difference decreases as the number of turbines increases. In Fig. 7 (b), we see that the real-time performance of the ANN improves significantly when GPU acceleration is enabled, placing it roughly on par with the TP model. Without GPU
 395 acceleration, however, the computational cost of the ANN is too high for applications such as WFLO, where many repeated evaluations would otherwise become prohibitively expensive.

The memory requirements of the considered models are similar, as shown in Fig. 7 (c), except for the GPU-accelerated ANN, which requires more memory. However, as all models remain below 1 GB of RAM, this remains well within an acceptable range. The more pressing concern is the availability of GPU resources.

400 3.3 WFLO study

A series of WFLO problems was solved using the optimization algorithms listed in Tab. 5 for each engineering model, both recalibrated and uncalibrated, as well as for the ANN model. All optimizations were executed on identical and exclusive nodes of the Sophia cluster (Technical University of Denmark, 2019b) to ensure comparable runtime conditions. As PyWake
 405 only. Table 8 summarizes the performance of different combinations of wake models and optimization algorithms. For each



algorithm within a given model, the random seed that achieved the highest expected AEP is reported. To aid interpretation, the AEP value for the best-performing algorithm in each model is highlighted in bold.

Table 8. AEP of the optimized layouts produced from the best-performing initial seeds. Results shown in bold indicate the optimization algorithm that achieved the highest AEP, selected for RANS validation.

Method	Low-fidelity AEP [GWh]			
	w/o Blockage		w/ Blockage	
	SLSQP	IPOPT	SGD	SGD
SG	3049	3050	3046	3058
SG [†]	2972	2966	2954	2969
TP	2974	2968	2956	2958
TP [†]	2750	2747	2721	2739
ANN	3041	3042	3040	3047

[†] Uncalibrated model

The results in Tab. 8 are separated into cases with and without blockage. Only the SGD algorithm was used for cases with blockage due to the substantially higher memory consumption of these models. The batch sampling inherent to SGD makes it computationally less expensive, making it the only approach feasible at the required wind-rose resolution. Consequently, SGD results are used throughout for cases with blockage. A visualization of the obtained layouts is available in Appendix B.

For AEP validation with RANS, one optimized layout was selected for each low-fidelity model, both with and without blockage, corresponding to the ten results marked with bold in Tab. 8. Table 9 presents the RANS AEP alongside the corresponding low-fidelity AEP and an accuracy estimate, together with key optimization metrics for assessing computational expense. The best-performing metrics in Tab. 9 are highlighted in bold. The inclusion of blockage did not improve the accuracy of any model, despite its greater physical fidelity; however, it substantially increased the optimization cost by factors of between $\sim 6\times$ and $\sim 20\times$. The ANN was an exception, where the blockage case was cheaper; this could be because the IPOPT algorithm, which succeeded in the case without blockage, more aggressively explores the loss landscape than the SGD, which is known to be resistant to local minima.

The TP model produced the layout with the highest RANS AEP, despite being the least accurate in predicting RANS AEP from its low-fidelity estimate, excluding uncalibrated counterparts. This outcome is somewhat unexpected given that the recalibrated models and ANN were tuned against RANS data, making RANS the designated ground-truth reference in this study. By contrast, the SG model reproduced the RANS AEP most accurately, followed closely by the ANN, with TP trailing significantly. Hence, the ability to reproduce the RANS AEP accurately was not correlated with the ability to find the highest RANS AEP layout through optimization in this study.

One possible explanation for this behaviour is that the issue arises as an artifact of the optimization process. To investigate this, we first examine whether, among the layouts previously studied in RANS, the layout obtained using each low-fidelity



Table 9. Comparison of RANS-validated AEP and optimization statistics for wind farm layouts generated using different wake models, evaluated with and without blockage. The best-performing values within each category are highlighted in bold.

Metric	w/o Blockage					w/ Blockage				
	SG	SG [†]	TP	TP [†]	ANN	SG*	SG ^{†*}	TP*	TP ^{†*}	ANN*
RANS AEP [GWh]	3083	3090	3093	3091	3080	3092	3091	3091	3088	3089
Low-fidelity AEP [GWh]	3050	2972	2974	2750	3042	3058	2969	2958	2739	3047
Low-fidelity Acc. [% of RANS]	98.92	96.18	96.16	88.98	98.75	98.9	96.04	95.71	88.7	98.67
Opt. alg.	IPOPT	SLSQP	SLSQP	SLSQP	IPOPT	SGD	SGD	SGD	SGD	SGD
Walltime [h]	4.513	0.4275	0.6851	0.5117	159.1	28.29	10.17	13.62	7.423	92.54

[†] Uncalibrated model, * Model with blockage effects included

model represents the best layout that the model could theoretically produce. To test this hypothesis, a cross-comparison is performed in which the AEP is calculated using each low-fidelity model for all layouts evaluated with RANS. In Fig. 8, the results of this cross-comparison are presented as a heatmap showing the change in AEP (Δ AEP) relative to the default optimized model–layout pair. Entries with positive Δ AEP in a given model row therefore indicate that a better layout exists compared to the one obtained during optimization. As illustrated in Fig. 8, significant improvements in AEP can be achieved for both the SG model and the ANN without blockage, whereas for the TP model, no better layout is available. In cases with blockage, however, there is a single better layout for the TP* model: the one generated by the TP model without blockage. In comparison, no better layout exists for the SG* model, and the improvement for the ANN* is so slight that it is not visible at the displayed decimal precision.

According to the data in Tab. 9, a trend appears indicating that the SG model and ANN produce more accurate AEP estimates than the TP model. To determine whether this trend holds more generally, the accuracy of the low-fidelity models is assessed by comparing their AEP estimates with the RANS AEP for all available layouts. This comparison is presented in Fig. 9, where additional aggregated metrics for each row have been included, along with a model podium to easily identify the models most closely matching RANS predictions for each layout. As shown in Fig. 9, the hierarchy of models in terms of AEP estimation accuracy places the TP model in third position. In contrast, the SG model and ANN compete for first place, with the SG model ultimately emerging as the most accurate. Both the SG model and the ANN are more advanced than the TP model. The SG model incorporates a transition from a top-hat shape in the near wake to a Gaussian profile in the far wake, addressing limitations of a fully Gaussian wake. In contrast, the ANN is substantially more expressive than the engineering models, with more than 28,000 tunable parameters compared to fewer than 10. While this analysis highlights differences in AEP estimation accuracy, the WFLO results reveal that accuracy alone does not guarantee optimal layouts. In fact, the recalibrated TP model produced the best layouts in our WFLO experiment, despite demonstrating the lowest accuracy among the calibrated models.

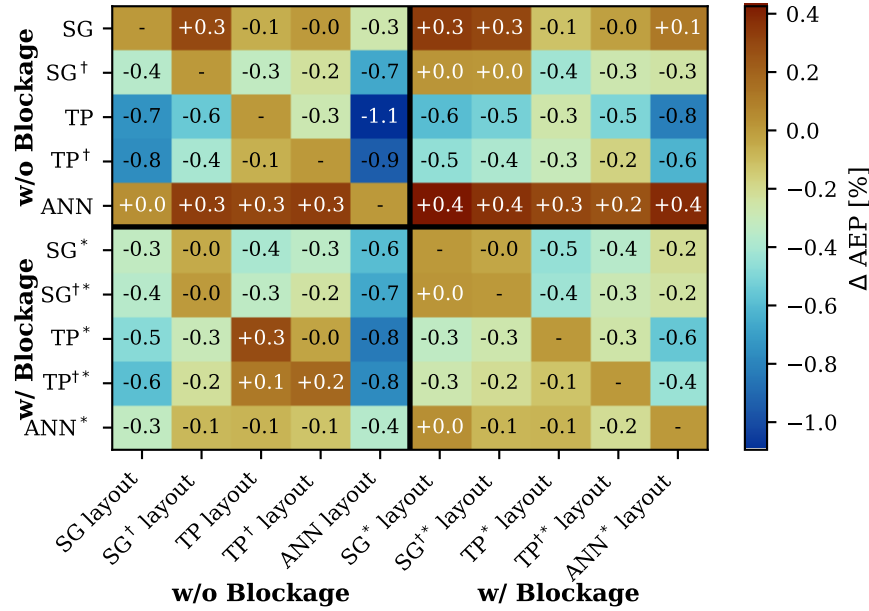


Figure 8. Cross-comparison of Δ AEP for layouts optimized by different low-fidelity models, showing changes relative to each model’s default layout. Each row corresponds to a distinct model used to compute AEP. Each column corresponds to a distinct layout generated by optimizing with a selected model.

450 The TP model achieved strong RANS AEP validated performance despite having lower AEP accuracy than the SG and ANN models. This was the first indication we observed that AEP accuracy, while important, is not the only critical factor in achieving an optimal layout. Further investigation across all RANS-validated layouts confirmed this pattern: the SG and ANN models consistently showed higher AEP accuracy, yet they did not yield the best layouts during WFLO. These observations are based on experiments with a limited number of initial random seeds, so while the results indicate that the TP model performs better, further experiments would strengthen this conclusion. That said, since computational resources are inherently limited in WFLO applications, this represents a realistic scenario for ANN-based optimization, and the TP model demonstrates better performance even under these practical constraints.

This insight raises two questions: why might a less accurate model be advantageous during optimization, and is this behavior systematic? Although a definitive answer remains elusive, we hypothesize that increased model fidelity in wake representations induces greater multimodality in the objective function geometry. As turbine-turbine interactions and the resulting flow become more complex, the relationship between turbine placement and AEP becomes correspondingly more complex. This complexity amplifies both the number and sharpness of local extrema, complicating the optimization problem and increasing the likelihood that gradient-based methods become trapped in local minima.

Multimodality has been examined in a recent paper by Poole (2025), which investigated its prevalence in circular farms and applied a smoothing strategy proposed by Thomas et al. (2017). Although limited to relatively small and simple farms, the

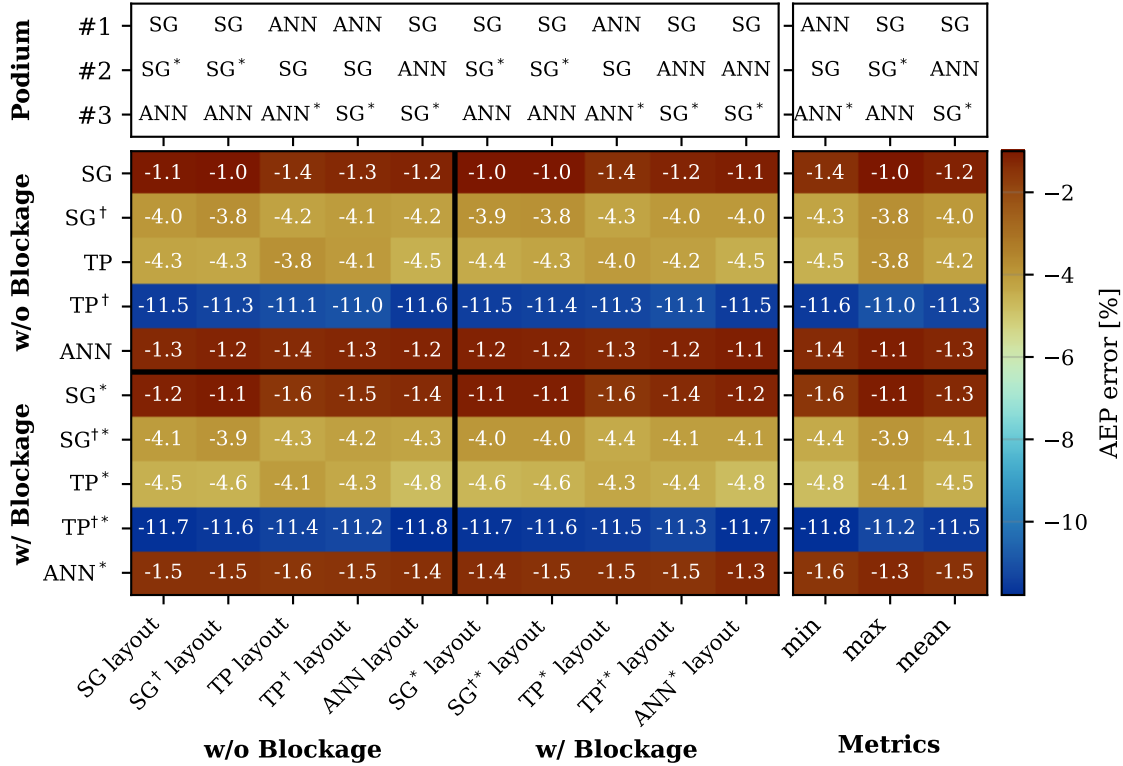


Figure 9. Comparison of AEP estimates from low-fidelity models against RANS for all layouts. Each row corresponds to a distinct model used to compute AEP. Each column corresponds to a distinct layout, which are the same layouts considered in Figure 8. The colors show the relative difference between the RANS-computed AEP and model-computed AEP associated with each layout. Podium rows indicate the top three models that most closely matched RANS predictions for each layout, while additional columns report aggregated metrics.

study establishes a correlation between farm size and multimodality. Simultaneously, they demonstrated that the mitigation strategy could reduce the modality of the objective function's geometry.

Earlier work by Stanley and Ning (2019) suggested reparameterizing wind farm layouts using the Boundary-Grid (BG) approach, which places turbines along the farm boundary. This assumption does not generally hold, as it ignores electrical infrastructure during optimization. Nonetheless, reducing multimodality by reducing the number of design variables remains a sound approach, as it can preserve AEP accuracy while making local minima easier to avoid.

An alternative approach, aligned with our hypothesis that complex wake modeling increases multimodality, is to simplify flow models. Thomas and Ning (2018) developed a wake model with an adaptable spread, enabling artificially smeared wakes during early optimization. Smearing of wakes reduces locally unwaked areas and lowers the number of local minima. As convergence nears, the wake spread is gradually removed, allowing final optimization with the original wake model. Another method is the FLOWERS analytical AEP model by LoCascio et al. (2024), which further simplifies classical assumptions by



considering a single turbine operating condition, constant wake expansion, a unified wind speed per directional bin, and a Fourier series approximation of wind directions. These simplifications enable an analytical AEP expression and significantly reduce the degree of multimodality.

480 To investigate this hypothesis, an experiment to visualize the multimodality of the objective function was conducted. Unfortunately, the admittedly limited experiment only showed a slight indication that the TP model produces lower multi-modality. Therefore, the experiment has been documented but moved to Appendix C as further studies are required to reach a definitive conclusion on the matter.

4 Conclusions

485 This study systematically validated an ANN-based surrogate model against two representative engineering models for wind farm flow simulation and WFLO. The main findings can be summarized as follows:

1. **Recalibration of engineering models:** Bayesian optimization applied to RANS-based single-wake data significantly improved the accuracy of both engineering models, with the most pronounced improvements observed in the near-wake region. These improvements are reflected in reduced RMSE and MAPE values compared to the original configurations,
490 indicating better alignment with RANS flow characteristics.
2. **Flow simulation performance:** Across all tested layouts and inflow conditions, the ANN surrogate consistently outperformed the engineering models in terms of $RMSE_{\Omega}$ and $MAPE_{\Omega}$. The ANN's ability to represent complex spatial flow structures enabled a higher fidelity flow representation. This improved accuracy comes at a higher computational cost than engineering models, though GPU acceleration mitigates this by enabling much faster real-world computational
495 time with a factor of $\sim 8\times$ for larger farms.
3. **WFLO results and optimization behavior:** Despite lower AEP-prediction accuracy, the recalibrated TP model produced layouts with the highest RANS-validated AEP. Conversely, the SG model and ANN surrogate achieved the most accurate AEP estimates but did not yield the best-performing layouts. This discrepancy suggests that higher model fidelity may introduce additional complexity in the optimization landscape, potentially making convergence for gradient-based algorithms, leading to the result that simpler models can occasionally yield layout of higher realized AEP.
500
4. **Impact of blockage modeling:** Including blockage effects did not improve AEP prediction accuracy or layout performance but substantially increased computational cost.

Further research should investigate the multimodality of the WFLO problem more thoroughly, particularly when using high-fidelity surrogates, including formally characterizing the convexity of the optimization landscapes associated with different wake models and conducting sensitivity analyses of optimal layout AEP with respect to wake model parameters.
505

Minimizing the impact of multimodality: Work focusing on multifidelity, employing simplified wake models in early iterations to reduce it initially, followed by high-fidelity surrogates for final convergence. An investigation into whether ANN-based



optimization can improve upon layouts generated by simpler models, such as the TP model, could further provide insight into the practical benefits of high-fidelity surrogates.

510 Finally, improving scalability with trust-region methods and extending validation to larger farms and broader inflow conditions will strengthen the applicability of surrogate-based optimization in real-world settings.



Code and data availability. The code associated with this publication is publicly available at <https://gitlab.windenergy.dtu.dk/surrogate-validation-study>. Because PyWakeEllipsys requires access to the closed-source Ellipsys3D solver, the flow data generated with it has been archived separately at <https://doi.org/10.5281/zenodo.18305003>. The RANS single-wake database used to calibrate the engineering wake models was published previously and is available at https://gitlab.windenergy.dtu.dk/TOPFARM/pywake_ranslut/-/tree/v0.3.

Author contributions. JPS, ES, and PER conceived the research; ES programmed the ANN to PyWake interface, ran the PyWake simulations and TopFarm optimizations, MPL simulated the RANS data, JQ performed the engineering model recalibration, JPS created layouts, performed post-processing, plotting, and analysis of results, JPS and PER supervised the work; JPS, wrote the original draft; and JPS, ES, JQ, MPL and PER reviewed the draft.

Competing interests. The authors declare that they have no competing interests.

Acknowledgements. In preparing this manuscript and the associated code, AI tools have been used for proofreading and code development and auto-completion: ChatGPT, Claude, Grammarly, Claude Code and Github co-pilot. Model training was performed on the Sophia cluster (Technical University of Denmark, 2019b).

This work is partially funded by the Joint Assessment of Models for Wind Energy project (Energy Technology Development and Demonstration Programme project number 134242-521856).



Appendix A: Flow study with blockage

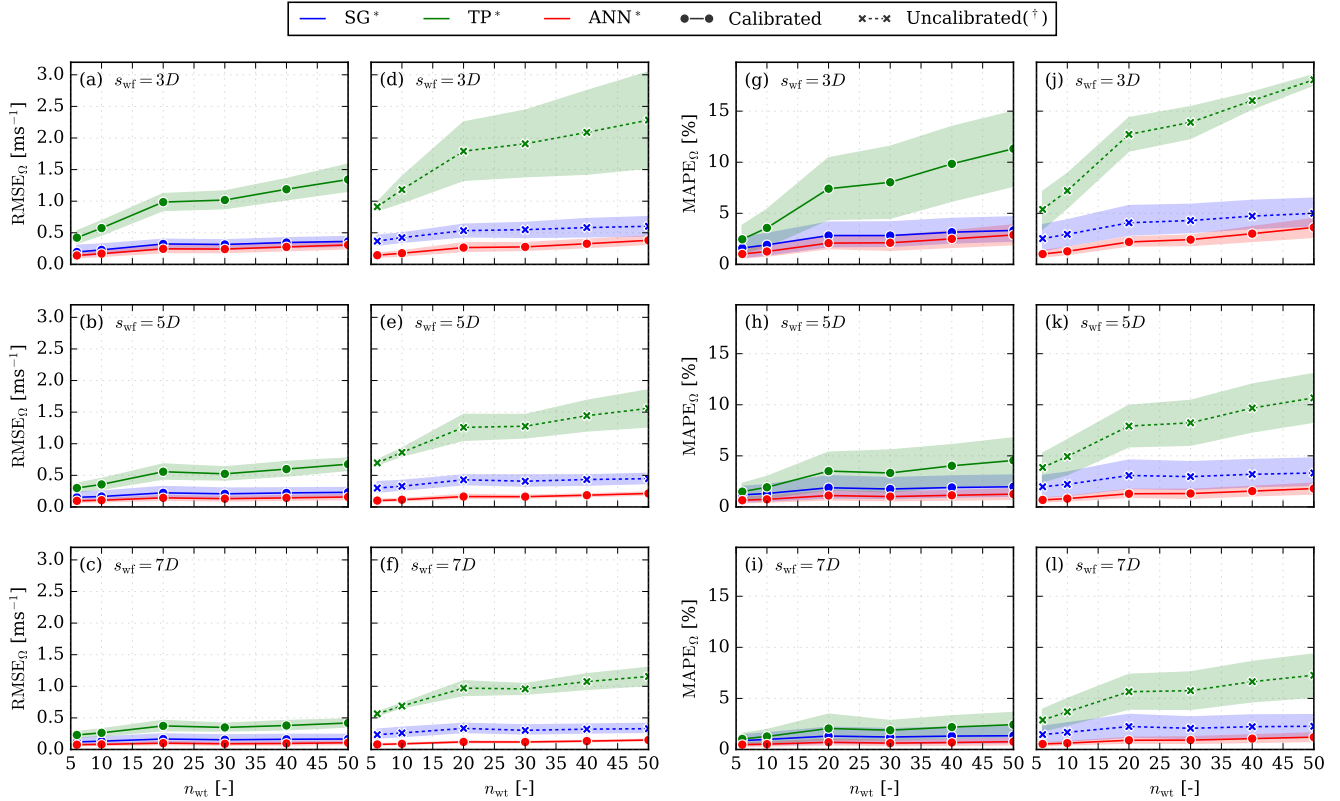


Figure A1. Flow study result metrics for an increasing number of wind turbines, with a blockage model. (a-c) RMSE with calibrated wake models, (d-f) RMSE with uncalibrated wake models, (g-i) MAPE with calibrated wake models, and (j-l) MAPE with uncalibrated wake models.

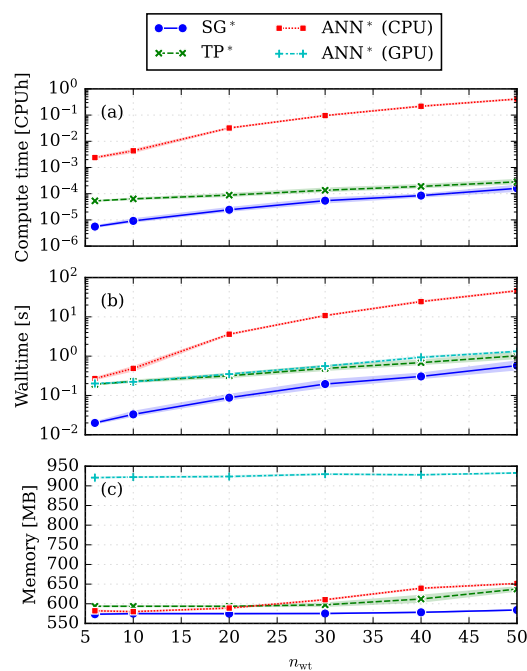


Figure A2. Average computational resource cost of a flow case for increasing wind-farm sizes with inclusion of blockage. (a) Relative cost in CPU hours, (b) real-world cost in walltime (seconds), and (c) Random Access Memory (RAM) consumption in MB.



Appendix B: Layouts obtained during WFLO

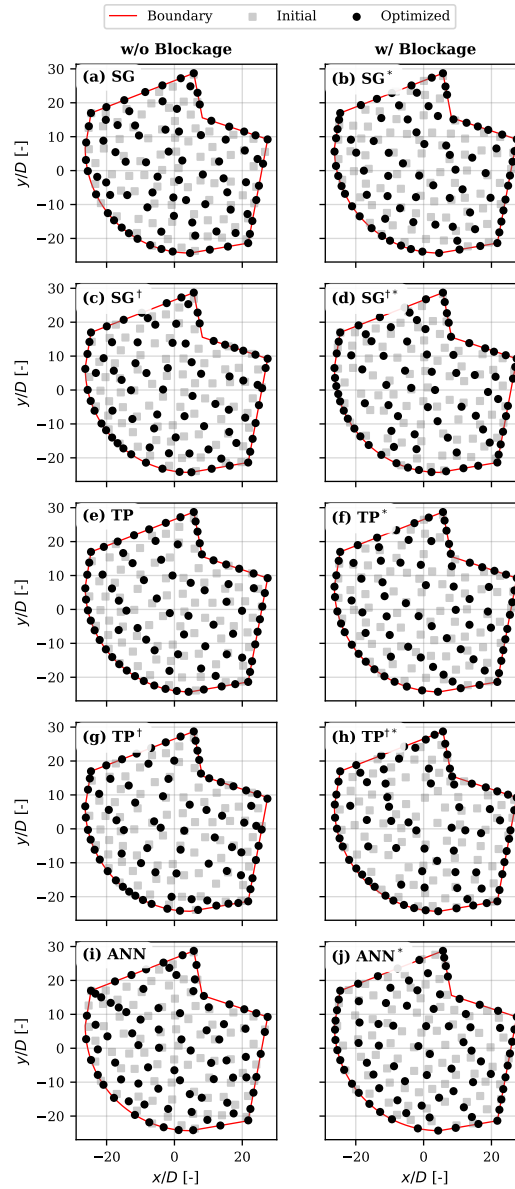


Figure B1. Initial and final layouts obtained through WFLO



Appendix C: Multi-modality experiment

We investigated the hypothesis, that the improved RANS AEP obtained using the TP model correlates with a lower degree of multi-modality in the objective function. To do so the SG, TP, and ANN models were employed to evaluate the improvement in AEP (ΔAEP) that could be achieved by adding a turbine to the best-performing layout obtained with the TP model. By systematically assessing the addition of the turbine at different locations throughout the farm. In practice, a grid was constructed with a density of one grid point per D , and the AEP improvement (ΔAEP) was then evaluated for every possible location the additional turbine could be placed in the grid.

Because the ranges vary significantly, which obscures the perceived smoothness of the objective function map, ΔAEP is min–max normalized independently for each observation, placing all the results within a range of 0 to 1:

$$\overline{\Delta\text{AEP}}(x, y) = \frac{\Delta\text{AEP}(x, y) - \min_{x, y}(\Delta\text{AEP})}{\max_{x, y}(\Delta\text{AEP})} \quad (\text{C1})$$

The resulting objective function maps, illustrating $\overline{\Delta\text{AEP}}$, are shown in Fig. C1. Additionally, two extra subplots are included, showing $\overline{\Delta\text{AEP}}$ at $y \approx \pm 10D$ for all three models in one plot at a time.

Figure C1 (a) to (c) illustrate that there is slightly less variability in the range of $\overline{\Delta\text{AEP}}$ for the TP model compared to the other two. In Fig. C1 (d) and (e), the cross-plane $\overline{\Delta\text{AEP}}$ is visualized. Here, the differences between the models are subtle: the TP model shows marginally smoother variations, with fewer extreme peaks and valleys, compared to SG and ANN, but the overall patterns remain broadly similar. As this can be difficult to see visually std (σ) values are displayed in Fig. C1 (d) and (e) to quantify the variability in each cross-section. Lower values of σ indicate smoother objective function landscapes. Here, the TP model consistently shows the lowest σ , supporting the observation of reduced multi-modality.

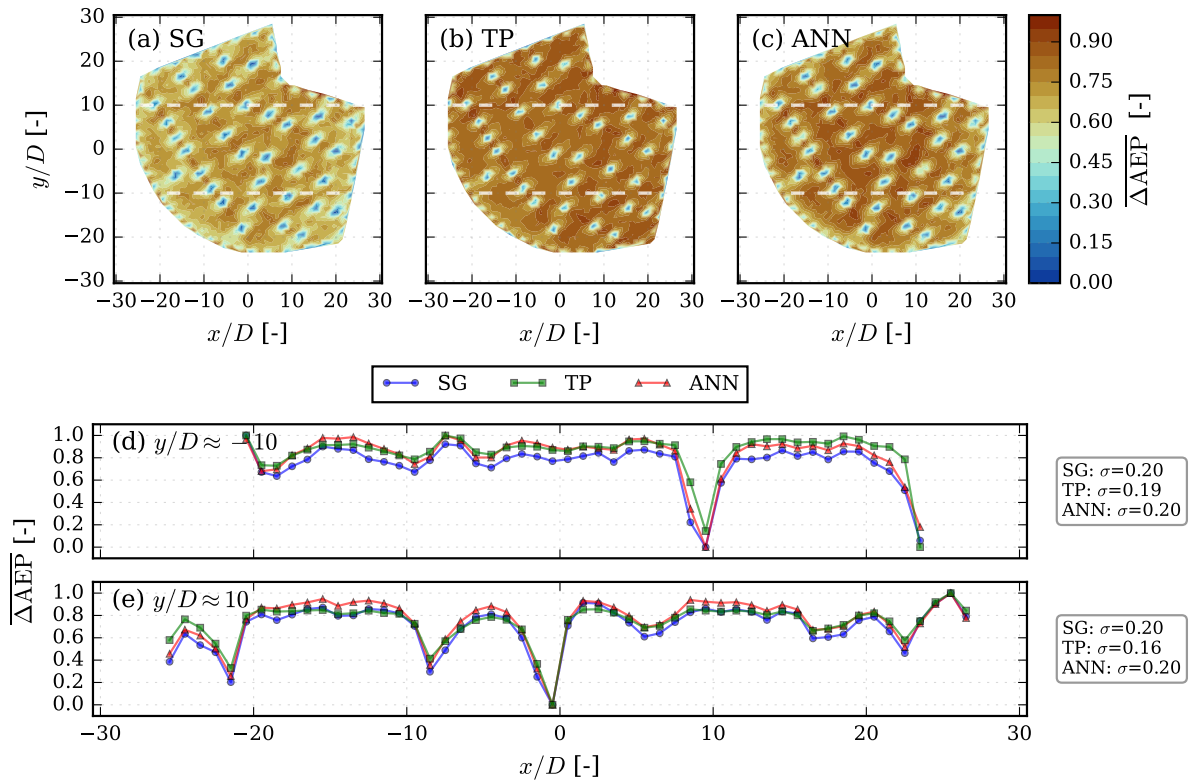


Figure C1. Objective function maps illustrating the normalized AEP improvement $\overline{\Delta AEP}$ for the calibrated low-fidelity models without blockage: (a) SG model, (b) TP model, (c) ANN model with a cross-plane profile for all models (d) at $y/D \approx -10$, and (e) at $y/D \approx 10$



References

- Anagnostopoulos, S. and Piggott, M.: Offshore wind farm wake modelling using deep feed forward neural networks for active yaw control and layout optimisation, *Journal of Physics: Conference Series*, 2151, 012 011, <https://doi.org/10.1088/1742-6596/2151/1/012011>, 2022.
- Andersen, S. J., Sørensen, J. N., Ivanell, S., and Mikkelsen, R. F.: Comparison of Engineering Wake Models with CFD Simulations, *Journal of Physics: Conference Series*, 524, 012 161, <https://doi.org/10.1088/1742-6596/524/1/012161>, 2014.
- Bak, C. ., Zahle, F. ., Bitsche, R. ., Kim, T. ., Yde, A. ., Henriksen, L. C., Hansen, M. H., Blasques, J. P. A. A., Gaunaa, M. ., and Natarajan, A.: The DTU 10-MW Reference Wind Turbine, in: *Danish Wind Power Research*, 2013.
- Baker, N. F., Stanley, A. P., Thomas, J. J., Ning, A., and Dykes, K.: Best Practices for Wake Model and Optimization Algorithm Selection in Wind Farm Layout Optimization, in: *AIAA Scitech 2019 Forum*, American Institute of Aeronautics and Astronautics, ISBN 978-1-62410-578-4, <https://doi.org/10.2514/6.2019-0540>, 2019.
- Bastankhah, M. and Porté-Agel, F.: A new analytical model for wind-turbine wakes, *Renewable Energy*, 70, 116–123, <https://doi.org/10.1016/j.renene.2014.01.002>, 2014.
- Blondel, F. and Cathelain, M.: An alternative form of the super-Gaussian wind turbine wake model, *Wind Energy Science*, 5, 1225–1236, <https://doi.org/10.5194/wes-5-1225-2020>, 2020.
- Bodini, N., Moriarty, P., Thedin, R., Doubrawa, P., Archer, C., Blaylock, M., Bottasso, C., Braunbehrens, R., Carmo, B., Cheung, L., Dubreuil, C., Floors, R., Herges, T., Houck, D., Kanjari, A., Kaul, C. M., Kelley, C., Li, R., Lundquist, J. K., Major, D., Nguyen, A. K., Optis, M., Parada, L. R. C., Peña, A., Quick, J., Ricarte, D., Radünz, W. C., Rai, R. K., Garcia Santiago, O. M., Schulte, J., Seim, K. S., van der Laan, M. P., Vimalakanthan, K., and Wise, A.: The AWAKEN wind farm benchmark, Part 2: Modeling results, *Wind Energy Science*, submitted, 2026.
- Bortolotti, P., Bay, C., Barter, G., Gaertner, E., Dykes, K., McWilliam, M., Friis-Møller, M., Pedersen, M. M., and Zahle, F.: System Modeling Frameworks for Wind Turbines and Plants: Review and Requirements Specifications, Tech. rep., National Renewable Energy Laboratory (NREL), <https://doi.org/10.2172/1868328>, 2022.
- Crespo, A. and Hernández, J.: Turbulence characteristics in wind-turbine wakes, *Journal of Wind Engineering and Industrial Aerodynamics*, 61, 71–85, 1996.
- Criado Risco, J., van der Laan, M. P., Pedersen, M. M., Meyer Forsting, A., and Réthoré, P.-E.: A RANS-based surrogate model for simulating wind turbine interaction, *Journal of Physics: Conference Series*, 2505, 012 016, <https://doi.org/10.1088/1742-6596/2505/1/012016>, 2023.
- Dar, A. S., Berg, J., Troldborg, N., and Patton, E. G.: On the self-similarity of wind turbine wakes in a complex terrain using large eddy simulation, *Wind Energy Science*, 4, 633–644, <https://doi.org/10.5194/wes-4-633-2019>, 2019.
- DTU Wind and Energy Systems: PyWakeEllipSys v5.3, https://topfarm.pages.windenergy.dtu.dk/cuttingedge/pywake/pywake_ellipsys, accessed: 27/05-25, 2023.
- Forsting, A. R. M., Diaz, G. P. N., Segalini, A., Andersen, S. J., and Ivanell, S.: On the accuracy of predicting wind-farm blockage, *Renewable Energy*, 214, 114–129, <https://doi.org/10.1016/j.renene.2023.05.129>, 2023.
- Frandsen, S.: Turbulence and turbulence-generated structural loading in wind turbine clusters - Risoe-R No. 1188(EN), Tech. rep., Forskningscenter Risoe, 2005.
- Gafoor, A., Boya, S. K., Jinka, R., Gupta, A., Tyagi, A., Sarkar, S., and Subramani, D. N.: A physics-informed neural network for turbulent wake simulations behind wind turbines, *Physics of Fluids*, 37, <https://doi.org/10.1063/5.0245113>, 2025.



- Gunn, K., Stock-Williams, C., Burke, M., Willden, R., Vogel, C., Hunter, W., Stallard, T., Robinson, N., and Schmidt, S. R.: Limitations to the validity of single wake superposition in wind farm yield assessment, *Journal of Physics: Conference Series*, 749, 012003, <https://doi.org/10.1088/1742-6596/749/1/012003>, 2016.
- 585 Herbert-Acero, J., Probst, O., Réthoré, P.-E., Larsen, G., and Castillo-Villar, K.: A Review of Methodological Approaches for the Design and Optimization of Wind Farms, *Energies*, 7, 6930–7016, <https://doi.org/10.3390/en7116930>, 2014.
- IEC: IEC 61400-1 ed. 3 Wind turbines-Part 1: Design requirements, International Standard, ISBN 978-2-8322-1971-3, <https://standards.iteh.ai/catalog/standards/sist/c065aa89-0f3c-4c31-8817-6bdec19e47eb/iec->, 2010.
- Jensen, N. O.: A note on wind generator interaction - RISØ-M-2411, Tech. rep., Risø National Laboratories, 1983.
- 590 Kainz, S., Quick, J., de Alencar, M. S., Moreno, S. S. P., Dykes, K., Bay, C., Zaijier, M. B., and Bortolotti, P.: The IEA Wind 740-10-MW Reference Offshore Wind Plants, Tech. rep., IEA Task 55, <https://github.com/IEAWindTask37/IEA-Wind-740-10-ROWP>, 2024.
- Kraft, D.: A software package for sequential quadratic programming. DFVLR-FB 88-28, Tech. rep., Institut für Dynamik der Flugsysteme, Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt (DFVLR), 1988.
- Li, H., Yang, Q., and Li, T.: Wind turbine wake prediction modelling based on transformer-mixed conditional generative adversarial network, *Energy*, 291, 130403, <https://doi.org/10.1016/j.energy.2024.130403>, 2024.
- 595 Liu, S., Li, Q., Lu, B., and He, J.: Prediction of offshore wind turbine wake and output power using large eddy simulation and convolutional neural network, *Energy Conversion and Management*, 324, 119326, <https://doi.org/10.1016/j.enconman.2024.119326>, 2025.
- LoCascio, M. J., Bay, C. J., Martínez-Tossas, L. A., Bastankhah, M., and Gorré, C.: FLOWERS AEP: An Analytical Model for Wind Farm Layout Optimization, *Wind Energy*, 27, 1563–1580, <https://doi.org/10.1002/we.2954>, 2024.
- 600 Madsen, H. A., Larsen, T. J., Pirrung, G. R., Li, A., and Zahle, F.: Implementation of the blade element momentum model on a polar grid and its aeroelastic load impact, *Wind Energy Science*, 5, 1–27, <https://doi.org/10.5194/wes-5-1-2020>, 2020.
- Michelsen, J. A.: Basis3D - a platform for development of multiblock PDE solvers., Tech. rep., DTU, 1992.
- Nogueira, F.: Bayesian Optimization: Open source constrained global optimization tool for Python, <https://github.com/bayesian-optimization/BayesianOptimization>, 2014.
- 605 Nygaard, N. G., Steen, S. T., Poulsen, L., and Pedersen, J. G.: Modelling cluster wakes and wind farm blockage, in: *Journal of Physics: Conference Series*, vol. 1618, IOP Publishing Ltd, ISSN 17426596, <https://doi.org/10.1088/1742-6596/1618/6/062072>, 2020.
- Nygaard, N. G., Pedersen, J. G., Hansen, S. D., and Krastins, P.: A Turbulence Optimized Park model, Tech. rep., Ørsted, <https://github.com/OrstedRD/TurbOPark/blob/main/TurbOPark%20description.pdf>, 2022.
- Panofsky, H. A. and Dutton, J. A.: Atmospheric turbulence. Models and methods for engineering applications, New York: Wiley, 1984.
- 610 Pedersen, M. M., Forsting, A. M., van der Laan, P., Riva, R., Romàn, L. A. A., Risco, J. C., Friis-Møller, M., Quick, J., Schøler, J. P., Rodrigues, R. V., Olsen, B. T., and Réthoré, P.-E.: PyWake 2.5.0: An open-source wind farm simulation tool, <https://gitlab.windenergy.dtu.dk/TOPFARM/PyWake>, 2023.
- Pedersen, M. M., Friis-Møller, M., Réthoré, P.-E., Simutis, E., Riva, R., Quick, J., Dimitrov, N. K., Rinker, J., and Dykes, K.: DTUWindEnergy/TopFarm2: Release of v2.6.1, <https://doi.org/10.5281/zenodo.17540961>, 2025.
- 615 Pish, F., Göçmen, T., and Laan, M. P. V. D.: Generalization of single wake surrogates for multiple and farm-farm wake analysis, *Journal of Physics: Conference Series*, 2767, 092058, <https://doi.org/10.1088/1742-6596/2767/9/092058>, 2024.
- Poole, D. J.: Characterization of Multimodality in Wind Farm Layout Optimization, *Energy Science & Engineering*, <https://doi.org/10.1002/ese3.70377>, 2025.



- Porté-Agel, F., Bastankhah, M., and Shamsoddin, S.: Wind-Turbine and Wind-Farm Flows: A Review, *Boundary-Layer Meteorology*, 174, 1–59, <https://doi.org/10.1007/s10546-019-00473-0>, 2020.
- Quick, J., Rethore, P.-E., Pedersen, M. M., Rodrigues, R. V., and Friis-Møller, M.: Stochastic gradient descent for wind farm optimization, *Wind Energy Science*, 8, 1235–1250, <https://doi.org/10.5194/wes-8-1235-2023>, 2023.
- Schøler, J. P., Riva, R., Andersen, S. J., Leon, J. P. M., van der Laan, M. P., Risco, J. C., and Réthoré, P.-E.: RANS-AD based ANN surrogate model for wind turbine wake deficits, *Journal of Physics: Conference Series*, 2505, 012 022, <https://doi.org/10.1088/1742-6596/2505/1/012022>, 2023.
- Schøler, J. P., Rosi, N., Quick, J., Riva, R., Andersen, S. J., Leon, J. P. M., Laan, M. P. V. D., and Réthoré, P.-E.: RANS wake surrogate: Impact of Physics Information in Neural Networks, *Journal of Physics: Conference Series*, 2767, 092 033, <https://doi.org/10.1088/1742-6596/2767/9/092033>, 2024.
- Sørensen, N. N.: General purpose flow solver applied to flow over hills, Ph.D. thesis, DTU, 1994.
- Stanley, A. P. J. and Ning, A.: Massive simplification of the wind farm layout optimization problem, *Wind Energy Science*, 4, 663–676, <https://doi.org/10.5194/wes-4-663-2019>, 2019.
- Sun, H. and Yang, H.: Wind farm layout and hub height optimization with a novel wake model, *Applied Energy*, 348, 121 554, <https://doi.org/10.1016/j.apenergy.2023.121554>, 2023.
- Sørensen, J. N., Nilsson, K., Ivanell, S., Asmuth, H., and Mikkelsen, R. F.: Analytical body forces in numerical actuator disc model of wind turbines, *Renewable Energy*, 147, 2259–2271, <https://doi.org/10.1016/j.renene.2019.09.134>, 2020.
- Technical University of Denmark: Sophia HPC Cluster, <https://doi.org/10.57940/FAFC-6M81>, 2019a.
- Technical University of Denmark: Sophia HPC Cluster, <https://doi.org/10.57940/FAFC-6M81>, 2019b.
- Thomas, J. J. and Ning, A.: A method for reducing multi-modality in the wind farm layout optimization problem, *Journal of Physics: Conference Series*, 1037, 042 012, <https://doi.org/10.1088/1742-6596/1037/4/042012>, 2018.
- Thomas, J. J., Gebraad, P. M., and Ning, A.: Improving the FLORIS wind plant model for compatibility with gradient-based optimization, *Wind Engineering*, 41, 313–329, <https://doi.org/10.1177/0309524X17722000>, 2017.
- Ti, Z., Deng, X. W., and Zhang, M.: Artificial Neural Networks based wake model for power prediction of wind farm, *Renewable Energy*, 172, 618–631, <https://doi.org/10.1016/j.renene.2021.03.030>, 2021.
- Troldborg, N. and Meyer Forsting, A. R.: A simple model of the wind turbine induction zone derived from numerical simulations, *Wind Energy*, 20, 2011–2020, <https://doi.org/10.1002/we.2137>, 2017.
- Troldborg, N., Sørensen, N., Réthoré, P.-E., and van der Laan, M.: A consistent method for finite volume discretization of body forces on collocated grids applied to flow through an actuator disk, *Computers Fluids*, 119, 197–203, <https://doi.org/10.1016/j.compfluid.2015.06.028>, 2015.
- van der Laan, M., Andersen, S., Kelly, M., and Baungaard, M.: Fluid scaling laws of idealized wind farm simulations, *Journal of Physics: Conference Series*, 1618, 062 018, <https://doi.org/10.1088/1742-6596/1618/6/062018>, 2020.
- van der Laan, M. P., Sørensen, N. N., Réthoré, P. E., Mann, J., Kelly, M. C., and Troldborg, N.: The $k-\varepsilon-f_p$ model applied to double wind turbine wakes using different actuator disk force methods, *Wind Energy*, 18, 2223–2240, <https://doi.org/10.1002/we.1816>, 2015a.
- van der Laan, M. P., Sørensen, N. N., Réthoré, P. E., Mann, J., Kelly, M. C., Troldborg, N., Schepers, J. G., and Machefaux, E.: An improved $k-\varepsilon$ model applied to a wind turbine wake in atmospheric turbulence, *Wind Energy*, 18, 889–907, <https://doi.org/10.1002/we.1736>, 2015b.



- 655 van der Laan, M. P., Andersen, S. J., and Réthoré, P.-E.: Brief communication: Wind-speed-independent actuator disk control for
 faster annual energy production calculations of wind farms using computational fluid dynamics, *Wind Energy Science*, 4, 645–651,
<https://doi.org/10.5194/wes-4-645-2019>, 2019.
- van der Laan, M. P., Andersen, S. J., Réthoré, P.-E., Baungaard, M., Sørensen, J. N., and Troldborg, N.: Faster wind farm AEP calculations
 with CFD using a generalized wind turbine model, *Journal of Physics: Conference Series*, 2265, 022 030, [https://doi.org/10.1088/1742-](https://doi.org/10.1088/1742-6596/2265/2/022030)
 660 6596/2265/2/022030, 2022.
- van der Laan, M. P., Meyer Forsting, A., and Réthoré, P.-E.: A Computational Fluid Dynamics surrogate model for wind turbine interaction
 including atmospheric stability, *Wind Energy Science Discussions*, 2026, 1–25, <https://doi.org/10.5194/wes-2025-287>, 2026.
- Virtanen, P., Gommers, R., Oliphant, T. E., Haberland, M., Reddy, T., Cournapeau, D., Burovski, E., Peterson, P., Weckesser, W., Bright, J.,
 van der Walt, S. J., Brett, M., Wilson, J., Millman, K. J., Mayorov, N., Nelson, A. R. J., Jones, E., Kern, R., Larson, E., Carey, C. J., Polat,
 665 İ., Feng, Y., Moore, E. W., VanderPlas, J., Laxalde, D., Perktold, J., Cimrman, R., Henriksen, I., Quintero, E. A., Harris, C. R., Archibald,
 A. M., Ribeiro, A. H., Pedregosa, F., van Mulbregt, P., and SciPy 1.0 Contributors: SciPy 1.0: Fundamental Algorithms for Scientific
 Computing in Python, *Nature Methods*, 17, 261–272, <https://doi.org/10.1038/s41592-019-0686-2>, 2020.
- Wächter, A. and Biegler, L. T.: On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming,
Mathematical Programming, 106, 25–57, <https://doi.org/10.1007/s10107-004-0559-y>, 2006.
- 670 Yang, K. and Deng, X.: Layout optimization for renovation of operational offshore wind farm based on machine learning wake model,
Journal of Wind Engineering and Industrial Aerodynamics, 232, 105 280, <https://doi.org/10.1016/j.jweia.2022.105280>, 2023.
- Yang, S., Deng, X., Ti, Z., Yan, B., and Yang, Q.: Cooperative yaw control of wind farm using a double-layer machine learning framework,
Renewable Energy, 193, 519–537, <https://doi.org/10.1016/j.renene.2022.04.104>, 2022.
- Zehtabiyani-Rezaie, N., Iosifidis, A., and Abkar, M.: Data-driven fluid mechanics of wind farms: A review, *Journal of Renewable and Sus-*
 675 *tainable Energy*, 14, <https://doi.org/10.1063/5.0091980>, 2022.