General Comments (Summary)

This paper presents a time-domain Rankine source method using a body-exact formulation to solve a forced oscillation wave-structure interaction known as a radiation problem. In the boundary value problem, linearized free-surface boundary conditions were used, but the body boundary condition is updated in time to correspond to the instantaneous wetted body surface. Unsteady and quadratic pressure terms from the Bernoulli equation were investigated by evaluating hydrodynamic loads, and radiated free-surface elevations were also investigated using three typical geometries: a truncated circular cylinder, spar platform, and semi-submersible floater. The presence of higher-harmonic components in the hydrodynamic loads and radiated waves was a main contribution in this paper due to the consideration of the instantaneous wetted body surface. As a result, the geometrical characteristics were discussed in the second- and third-harmonic components by comparing them with classical linear computations under the mean wetted body surface.

Overall, the paper addresses interesting topics that are actively discussed within the community. However, the mathematical proof and its supporting arguments are to be improved. The full list of comments is written in the following parts. In short, to strengthen this study, further validation against existing nonlinear methods, experimental measurements, or high-fidelity solvers (such as CFD) is necessary. Therefore, the paper cannot be accepted in its current form and requires major revisions.

Specific Comments

- The literature review needs improvement. In the introduction, the authors repeat the same papers and focus too narrowly, which should be significantly improved. Furthermore, an additional table or figure showing different approaches (linear, weakly nonlinear, nonlinear) and limitations in terms of physics (first harmonic, second harmonic, higher harmonics) with computational efficiency would be beneficial to show where this methodology stands as the state of the art (for example, Table 2. Summary of prediction methods for added resistance due to waves in ITTC 7.5-02-07-02.8).
- Definition of body exact. From the reviewer's understanding, the body-exact mean in this paper is the instantaneous wetted body surface with respect to the mean free-surface level (z=0). However, in Section 2 (Methodology), the reader may be confused because the exact wetted body surface is either cut by the disturbed wave field (radiated waves) or by the mean free-surface level (z=0). This should be clearly mentioned in the methodology.
- The body-exact formulation needs to be further detailed and clearly explained. The authors discuss higher-harmonic forces from the unsteady and quadratic pressure terms in Bernoulli's equation, but it is difficult to see where these higher-order forces come from. Since the free-surface boundary condition is linearized at $z = 0$ under the assumption of small wave amplitude, and the imposed motion is also linear, the perturbed velocity potential remains linear. As a result, the unsteady pressure should be first order. From the formulation, the higher-harmonic components appear 1) from the quadratic term (Pq) or 2) when the pressure components (Pu and Pq) are integrated along the wetted body surface, especially for geometries with complexity such as bow flare or a stern on a ship-shaped structure.

  However, when the authors present the results for the truncated vertical circular cylinder with a 1 m radius and a 5 m draft, the second harmonic is significantly pronounced in the vertical hydrodynamic forces under forced heave motions. In particular, for the pure heaving cylinder, the vertical hydrodynamic forces come from integration over the bottom surface only, as the other parts cannot contribute to the z-direction because their normals point in the radial direction. Therefore, the reviewer would expect that only the first harmonic (linear component) plays a major role and that there are no non-linear contributions to the vertical forces in this case.
- Mathematical formulations are needed to support the presence of higher harmonics, along with validation against an experimental database (from the literature, if available) or CFD. An investigation of energy conservation needs to be performed (see Faltinsen - Sea loads 1993 in energy relation equation 3.19) in order to proof the robustness of the method.

Technical Corrections

- Equation (1): The governing equation (Laplace equation) needs to be added for the Boundary Value Problems.
- Equation (1): The definition of body motion U should be presented here.
- Line 108: The deep-water assumption should be stated here.
- Equation (2): Add a graphical image presenting the boundaries and describing the damping zone; this would be beneficial for the reader to understand.
- Line 124: The definition of the wave number, k, is missing.
- Equation (3): This equation can be further developed with a perturbation series with respect to the order of ka (See S. Malenica and B. Molin, Third-harmonic wave diffraction by a vertical cylinder, JFM, 1995).
- Equation (4): The hydrodynamic forces can be further developed to see different orders?
- Line 140: Describe the Dirichlet condition more precisely, as the authors did for the Neumann condition (body boundary).
- Line 155: The term $\rho g z$ in the Bernoulli equation needs special attention. In the linear theory, this term is considered as the hydrostatic force and put into the restoring term. However, in the nonlinear theory, this "z" is carefully treated since "z" is the wave elevation η, and η is a function of perturbed velocity potential from the dynamic free surface boundary condition (see the following reference although it is for the diffraction theory: third-order formulation written by S. Malenica and B. Molin, Third-harmonic wave diffraction by a vertical cylinder, JFM, 1995).
- Line 166: Meaning of the moving waterline – precisely describe the moving water line at z =0 or z=eta.
- Section 2.3: A flow chart would be beneficial for BEMUseRS with various computation steps coupled with BEMUse in the frequency domain.
- Section 3: Perform a validation study using existing experimental measurements to confirm the higher-harmonic findings. The reviewer believes there are many existing experimental or high-fidelity numerical models to compare your findings, particularly for the truncated circular cylinder.
- Section 4: Further verification is necessary. Since the circular cylinder case is the representative case, additional sensitivity studies with various cylinder diameters and depths would be beneficial to see how the higher harmonic loads change.
- Furthermore, please also check how the pressure field on the bottom and the radiated free-surface elevation near the oscillating body should be presented visually to better understand the physics, not only the force and wave time series.
- Figure 6: typo - "The last two periods are shown"; three oscillation periods are shown.
- Figure 7: In the hydrodynamic spectra of Fx and My from pure surge and pitch oscillations, respectively, the third-harmonic components are about half the order of the first harmonic (e.g., max motion amplitude case A = 0.4 m or 12 degrees). However, this can't be seen in the time histories in Figure 6. Could you explain why?
- Figure 10. Why are the dashed lines (from the power law) shorter than the motion amplitude range?
- Section 5.2.1: The authors present the radiated wave field only for the spar platform. If the authors would like to present one case of radiated waves, the cylinder case would be fitted better, as this is the representative case.
- Conclusion: The physical meaning of the higher-harmonic components from the current methods needs to be discussed. In general, the first harmonic (or linear) component serves as added mass and wave-radiated damping. What does it mean of higher harmonics? How can the higher-harmonic components be treated in the state-of-the art engineering design tools?